Parabola Volume 14, Issue 3 (1978)

PROBLEM SECTION

You are invited to submit solutions to one or more of these problems. Solutions should be written with a pen, and the solution to each problem should start on a new page. Each page should have on it your name, school and year. Solutions will be published in the issue after next.

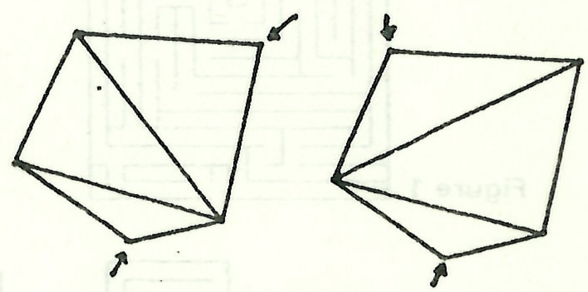
393. Show that if n is any integer greater than 2, of the fractions $1/n, 2/n, 3/n, \ldots, (n-1)/n$ an even number are in lowest terms.

394. Prove that, if $a^2 + b^2 = x^2 + y^2 = 1$, then $ax + by \leqslant 1$.

395. A polygon is said to be triangulated when diagonals, no two of which cross, are drawn cutting the polygon into triangles. A polygon other than a triangle can be triangulated in more than one way.

(a) Show that a triangulated n-gon is always cut into $n-2$ triangles by $n-3$ diagonals.

(b) Show that there are at least two vertices of a triangulated polygon each of which lies in a single triangle.

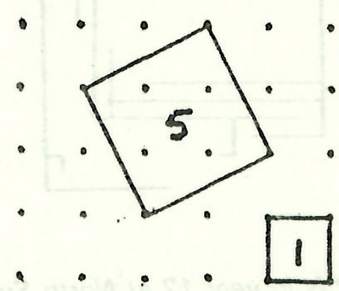


396. Find a 10 digit number whose first digit tells the number of zeros which appear in it, whose second digit tells the number of ones, and so on; (thus the tenth digit tells the number of nines in the number). Is there another such number?

397. The smallest square on a peg-board has unit area. The figure shows how to construct squares of area 1 and 5 using pegs and rubber bands.

(a) Show how to construct squares of areas 8 and 10.

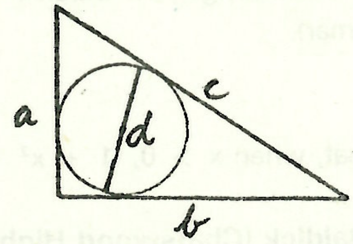
(b) Prove that it is not possible to construct a square of area $4n+3$ where n is an integer.

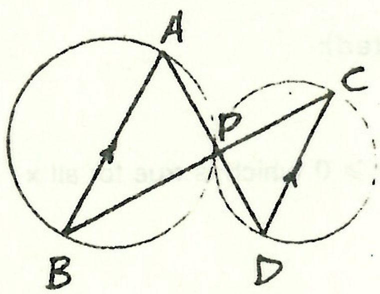


398. Show that it is impossible to construct an equilateral triangle on the pegboard in question 397 using 3 pegs and a rubber band.

399. Show how to construct an equilateral triangle by folding a single (rectangular) sheet of paper. No rulers, compasses or separate sheets for measuring are to be used.

400. Show that the diameter d of the inscribed circle of a right triangle of legs a, b and hypotenuse c satisfies $d = a + b - c$.





401. Let AB and CD be parallel diameters of two circles which touch at P. Show that the lines BC and AD intersect at P.

402. Consider 5 points in space such that each pair is not more than 1 cm apart. What is the greatest number of pairs which can be exactly 1 cm apart? Prove your answer. (If there are 4 points there can be as many as six pairs exactly 1 cm apart — take the four points at the vertices of a regular tetrahedron).

403. Given any set of ten distinct positive integers each less than 100 show that there are two subsets of this set having no elements in common such that the sums of the numbers in the subsets are equal.

404. The number 1234567 is not divisible by 11, but 3746512 is. How many different multiples of 11 can be obtained by appropriately ordering these digits?

Solutions to Problems from Vol. 14 No. 1

369. Find a five digit number which when divided by 4 yields another 5 digit number using the same 5 digits but in the opposite order.

Solution: G.J. Chappell (Kepnock High) writes:

Let the number be $y = ABCDE$ and $x = EDCBA$ so $y = 4x$. Now $y < 100{,}000$ so $x < 25{,}000$ so $E = 0, 1$ or 2 but y is even so $E = 0$ or 2, but x has five digits ($E \neq 0$) so $E = 2$.
Since $y = 4x$, $E = 4 \times A \pmod{10}$ so $A = 3$ or 8, but $x \geqslant 20{,}000$ so $y \geqslant 80{,}000$ so $A = 8$.
But $y \leqslant 89{,}992$ so $x \leqslant 22{,}498$; $x \geqslant 20{,}008$ so $D = 0, 1$ or 2. Since $y = 4x$, $D = 4 \times B + 3 \pmod{10}$ which is odd so $D = 1$.