TRIANGULATION

D. McGrath*

Here is a pretty problem to exercise your geometrical skills.

Take any scalene triangle ABC. Now from the point C mark off segments CC′ and CC″ along CA and CB with length equal to the side AB. (See figure 1.) Similarly, mark off segments BB′ and BB″ along BC and BA with length AC and segments AA′ and AA″ along AB and AC with length BC. (See figure 2. It will be necessary to produce some of the sides of the triangle as shown.) Finally, mark in the following midpoints; P is the midpoint of BC, Q is the midpoint of AC, R is the midpoint of AB, S is the midpoint of AB″, T is the midpoint of AC′, V is the midpoint of BC″, W is the midpoint of BA′, Y is the midpoint of CA″ and Z is the midpoint of CB′.

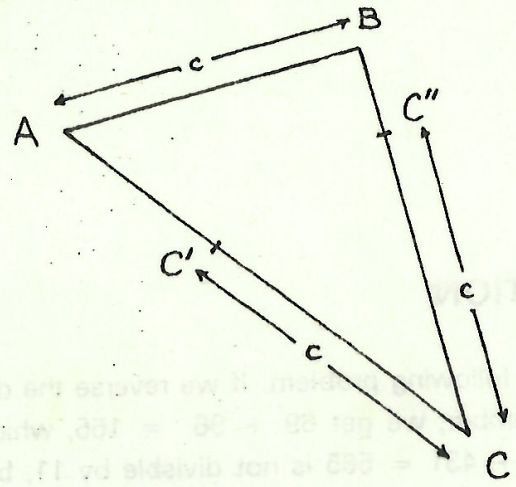

Figure 1

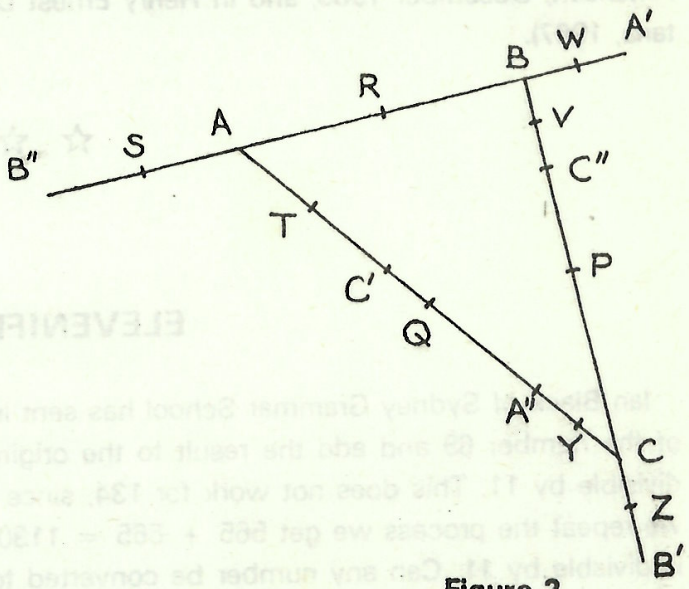

Figure 2

Prove the following coincidences (indicated in Figure 3):

(1a) The points S, T and P are collinear,

(1b) The points W, V and Q are collinear,

(1c) The points Z, Y and R are collinear,

and, wonder of wonders,

(2) The lines SP, WQ and ZR are concurrent.

If you don't believe it, draw the figure yourself.

** David McGrath is in year 10 at The King's School.*

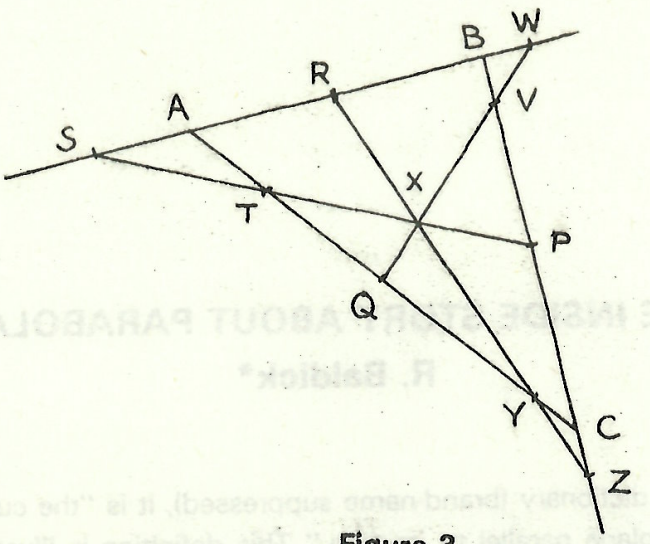

Figure 3

David McGrath's most ingenious solution and any other solutions received from our readers will appear in a subsequent issue of Parabola. If you have a solution or an extension of the result write in and tell us about it. Post early to avoid disappointment.

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SOME SUM

Ian Black of Sydney Grammar School has sent us the following palindromic equations:

$$12 + 32 + 43 + 56 + 67 + 87 = 78 + 76 + 65 + 34 + 23 + 21$$
$$12^2 + 32^2 + 43^2 + 56^2 + 67^2 + 87^2 = 78^2 + 76^2 + 65^2 + 34^2 + 23^2 + 21^2$$
$$12^3 + 32^3 + 43^3 + 56^3 + 67^3 + 87^3 = 78^3 + 76^3 + 65^3 + 34^3 + 23^3 + 21^3.$$

What is behind these extraordinary facts?

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