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PALINDROMES, SEMORDNILAP

Here, at last, are the results of the palindrome competition announced in Parabola, Volume 13, Number 3. A very good entry was sent in by Ian Black of Sydney Grammar School; he wins the prize of a pocket calculator donated by Sanyo. The runners up were Philip Bos of Sydney High School and Terry Jones of Cranbrook School. This report has been made up from the letters sent in by our three contestants.

A palindromic number is a positive integer which reads the same backwards and forwards, for example 11, 131, 36763. (See Parabola, Volume 12, Number 3.) Our competition was concerned with three questions posed in Parabola, Volume 13, Number 1.

(1) *Palindromic numbers and divisibility by 11.* We need the test for divisibility by 11: the number N = abcd...e is divisible by 11 if and only if the alternating sum of its digits S = a−b+c−d+ ... ±e is divisible by 11. (For example, if N = 618354, then S = 6−1+8−3+5−4 = 11, so N is divisible by 11. Of course, the digits a,b,c,... come from writing N in base 10.) Now, if N = abcd...dcba is a palindrome with an even number of digits then S = a−b+c−d+ ...+d−c+b−a = 0, so N must be divisible by 11. It is easy to find palindromes with any odd number of digits which are divisible by 11, for example 121, 12221, 1222221,..., and others which are not divisible by 11, for example 131, 13331, 1333331,.... We could consider palindromes in any base and not just in base 10: a palindrome in base b with an even number of digits must be divisible by $b+1$.

(2) *Palindromification.* If we reverse the digits of the number 134 and add the result to the original number, we get 134 + 431 = 565, a palindrome. This does not work for 517, since 517 + 715 = 1232 is not a palindrome, but if we repeat the process we get 1232 + 2321 = 3553 which is a palindrome. Other numbers may require more steps. For example, starting with 69, we have 69 + 96 = 165, 165 + 561 = 726, 726 + 627 = 1353, 1353 + 3531 = 4884 and reach a palindrome in four steps. This suggests that if we continue long enough, any number can be converted into a palindrome by this process. However, this has been neither proved, nor disproved. There is some counter-evidence. Even after several thousand steps, the number 196 has not been converted into a palindrome. This example, and others, indicate that there is no simple way of predicting the number of steps required to produce a palindrome. If we change from base 10 to a base which is a power of 2, then we can find numbers which cannot be converted into palindromes. For example, in base 2, the number 10110 (= 22 in base 10) gives the following pattern:

$$10110 \xrightarrow{\text{2 steps}} 10\underline{1}01\underline{00} \xrightarrow{\text{2 steps}} 10\underline{11}01\underline{00} \xrightarrow{\text{2 steps}} 10\underline{11}01\underline{000} \xrightarrow{\text{2 steps}} 10\underline{111}01\underline{000} \longrightarrow \ldots,$$

and so on. In base 4, the number 10332 gives

$$10332 \xrightarrow{\text{2 steps}} 10\underline{3}23\underline{00} \xrightarrow{\text{3 steps}} 10\underline{33}23\underline{00} \xrightarrow{\text{3 steps}} 10\underline{33}23\underline{000} \longrightarrow \ldots,$$

You can prove that these patterns persist by a somewhat messy induction argument. There is a similar example for any base which is a power of 2.

It does not seem to be possible to pick out which numbers will yield a palindrome in one step. Certainly, if there are no "carry ones" in the addition, then we get a palindrome in one step. (For example, 143256 + 652341 = 795597.) But, other numbers also have this property, for example 29 + 92 = 121.

(3) *Palindromic products.* We want more examples like 23 × 64 = 46 × 32. It is easy enough to find all the two-digit numbers which can be multiplied in this way. The property we need is that $(10a + b)(10c + d) = (10d + c)(10b + a)$, that is $ac = bd$. So it is a matter of finding all numbers which can be factorised in two different ways as a product of two digits between 1 and 9. There are 14 different possibilities, which you may care to find. Examples of this phenomenon involving larger numbers can be constructed at will, but it would be much more difficult to find all of them. For example, we may start with the obvious equation

$$121314 \times 413121 = 121314 \times 413121,$$

and multiply two of the numbers by 2, giving

$$121314 \times 826242 = 242628 \times 413121,$$

which is a palindromic product.

More information about palindromes, as well as plenty of difficult problems, can be found in Martin Gardner's column in Scientific American, Volume 223, August 1970, pages 110-112.

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WATCH YOUR P's AND Q's
(Big Brother is watching you)

The Americans, being Americans, have just staged the Ninth Annual North American Computer Chess Tournament, where twelve computer programs battled for Caissa's favours. Two of the contestants, including the winner, were micro-computers — they were carried into the playing area and plugged in at the start of play; the other contestants were mere robots, receiving their instructions by telephony from a big computer at home. Put all this together and you have six desks smothered in electronic gadgetry, making Bobby Fischer's fees look like chickenfeed.

The most interesting game was the contest between "Chess 4.7", the reigning giant, and "Belle", the winner of the tournament. We give the moves below. "Belle" runs on two processes and, as soon as it has decided on its own move, it forks off a new process to analyse the consequence of what it predicts its opponent's response will be. The climax of the game came when, after a very long pause during which "Chess 4.7" analysed more than two million positions, it announced its move, and "Belle" responded immediately. That's oneupmanship with a vengeance. Just over ten years ago, the noted British better and International Master, David Levy, wagered $5000 that no computer chess program would beat him within ten years and he successfully collected his bet last August. This game may explain why he has not renewed his bet, though the programmers still have a long way to go to reach the top.

"Belle" is the brainchild of Ken Thompson and Joe Condon of Bell Telephone Laboratories. (Did you guess that?) The program analyses as many as 6000 moves per second, but speed is only a small part of the story. Not even a computer can analyse all the possible moves in a complicated position, and great programming sophistication is needed in order to evaluate a position and plan a winning strategy. One by-product of Ken Thompson's research is the operating system UNIX which he designed specifically to serve the needs of computer chess. It is used in the computer system at the University of New South Wales and it may even play chess with you if you know the magic password.

Will the computer ever teach us anything about chess? In any case, we venture to say that the machine cannot even be admitted to the novice class until it has developed an addiction to tobacco and an insatiable appetite for yoghurt.

Here is the game mentioned above: White — Belle, Black — Chess 4.7.

1 P–K4 N–QB3 2 P–Q4 P–Q4 3 N–QB3 P–K3 4 N–B3 B–N5 5 P–K5 KN–K2 6 B–Q2 N–B4 7 N–K2 B–K2 8 P–B3 O–O 9 N–B4 P–KB3 10 B–Q3 (White could take the initiative by playing 10 P–KN4!? first) 10 ...PxP 11 PxP P–KN4!? (Imaginative, but risky) 12 P–KN4 (Virtually forced, as the White KP is threatened after Black plays P–N5) 12 ...N–N2 13 N–N2 P–N3 14 Q–K2 B–N2 15 R–KN1 (15 P–KR4 looks stronger, since the line 15 ...RxN 16 BxRP ch does not work) 15...P–QR4 16 P–QR4 K–R1 17 P–R3 K–N1 (Has the spirit of Petrosian got into the machines?) 18 R–R1 P–R3 19 P–R4 (19 N–Q4 looks better) 19 ...P–Q5! 20 PxNP N–N5!! 21 PxRP NxB ch?? (Stronger is 21...PxBP! White now wins material by force) 22 QxN PxP 23 Q–N6! PxB ch 24 NxQP R–B2 25 PxN RxNP 26 QxKP ch R–B2 27 Q–R6 R–N2 28 Q–R8 ch K–B2 29 P–K6 ch! KxP 30 QxR BxN 31 R–R6 ch K–Q2 32 O–O–O! B–Q4 33 N–K4 K–B1 34 R–R8 BxN 35 R(Q1)xQ ch BxR 36 Q–K7 K–N2 37 QxB ch K–R2 38 R–N8 R–N1 39 P–N5 B–K2 40 RxR BxNP ch 41 P–B4 BxP ch 42 QxB KxR 43 K–Q2 K–N2 44 K–Q3 K–B1 45 P–QN4 PxP 46 QxP K–Q2 47 Q–N5 ch K–Q1 48 K–K4 Black resigns.