OLD MAN RIVER

Take two twenty-cent coins A and B. (Nothing is safe from inflation: once upon a time, not so long ago, this trick could be done with two pennies.) If the coin B is kept fixed and A is rolled round B without slipping, how many revolutions will A make about its centre before it returns to its original position? (See Figure 1.) Well, let's see: the circumferences of A and B are equal and the circumference of A is laid out once along that of B, so A must make one revolution. Is this correct?

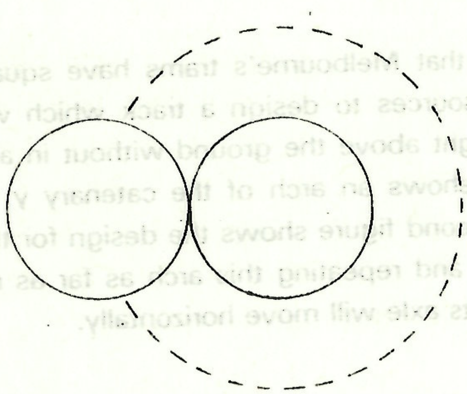

Figure 1

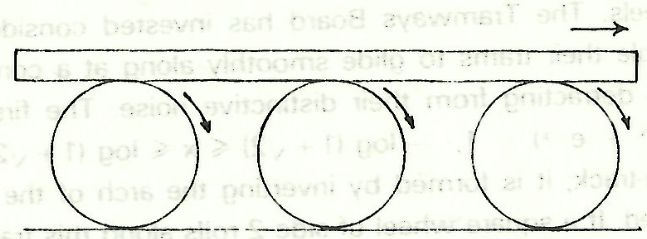

Figure 2

A heavy slab can be moved easily by using a number of rollers. (See Figure 2.) If the circumference of each roller is 1 foot, how far forward will the slab have moved when the rollers have made one revolution? Well, the distance moved must be equal to the circumference of the rollers, that is 1 foot. Is this correct?

Figure 3 shows a little conundrum known as Aristotle's wheel. The large circle rolls without slipping along the line PQ and makes one revolution in going from P to Q. The distance PQ is therefore equal to the circumference of the large circle. But the small circle, fixed to the large one, has also made one revolution, so that the distance RS is equal to the circumference of the small circle. Since RS is equal to PQ, we conclude that the circumferences of the two circles are equal! Surely, this isn't correct. Can you explain the paradox? Galileo considered this puzzle and you can find his explanation in his "Dialogues concerning two new sciences" (Dover reprint, 1954, page 20 onwards).

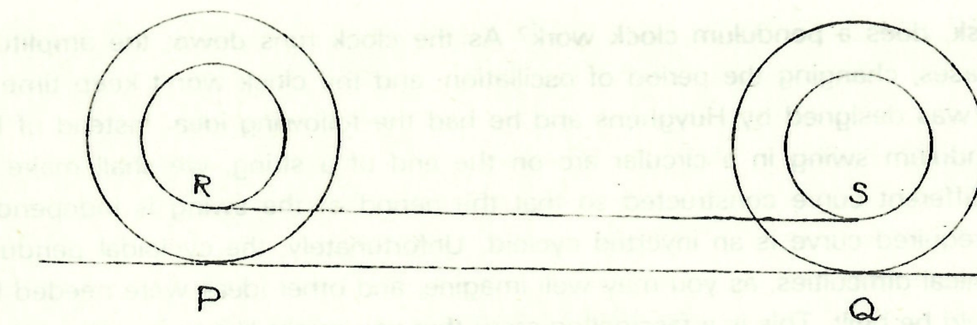

Figure 3

Did you know that no matter how fast a train is moving forwards, there are certain parts of the train which are moving backwards? (This has nothing whatever to do with the size of the railway deficit.)

The most famous rolling problems all lead to the cycloid. This is the curve traced out by a fixed point P on the circumference of a circle as the circle rolls without slipping along a straight line. (See Figure 4.) The cycloid has been called the "Helen of Geometers." Now this may be because it looks so beautiful, but it seems more likely that the real reason is that the cycloid caused so much strife among the mathematicians of the seventeenth century.

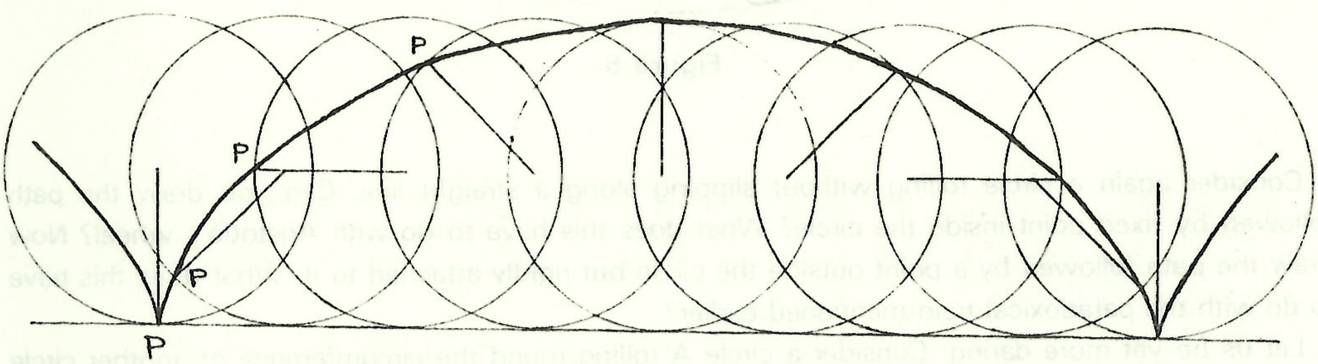

Figure 4

One knotty question is to find the area of one arch of a cycloid between the cycloid and its base line. Galileo tried to find this area by weighing a piece of metal cut in the shape of the arch and he guessed that the ratio of the area of the arch to the area of the rolling circle was π to 1. Try this experiment by cutting a cycloid out of heavy cardboard. Do you think Galileo's guess is correct? Attempts to find the exact area led to important advances on the road to the integral calculus.

Consider a simple pendulum which consists of a mass on the end of a string of length ℓ. Under the usual simplifying assumptions, the mass swings backwards and forwards with period $\sqrt{(g/\ell)}$, where g is a certain constant called the gravitational acceleration. However, this is only an approximation and the period of oscillation actually depends on the amplitude of the swing as well. So how, you will ask, does a pendulum clock work? As the clock runs down, the amplitude of the pendulum decreases, changing the period of oscillation, and the clock won't keep time. The first pendulum clock was designed by Huyghens and he had the following idea. Instead of letting the mass of the pendulum swing in a circular arc on the end of a string, we shall make the mass swing along a different curve constructed so that the period of the swing is independent of its amplitude. The required curve is an inverted cycloid. Unfortunately, the cycloidal pendulum clock runs into mechanical difficulties, as you may well imagine, and other ideas were needed before accurate clocks could be built. This is a fascinating story that you might like to investigate.

Undoubtedly the most famous appearance of the cycloid was Jacob Bernoulli's brachistochrone problem. The problem is to find the path down which a particle will roll from one given point to another not directly below it in the shortest time. (See Figure 5.) (Brachistochrone is a Greek hybrid meaning shortest time.) The answer is not the straight line path, though it gives the shortest distance, nor is it, as Galileo thought, the arc of the circle. It is, in fact, part of an inverted cycloid.

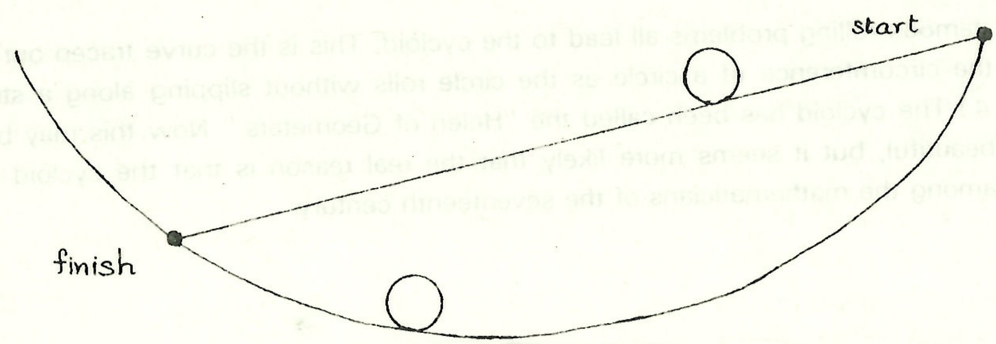

Figure 5

Consider again a circle rolling without slipping along a straight line. Can you draw the path followed by fixed point inside the circle? What does this have to do with Aristotle's wheel? Now draw the path followed by a point outside the circle but rigidly attached to it. What does this have to do with the paradoxical train mentioned earlier?

Let us be yet more daring. Consider a circle A rolling round the circumference of another circle B. What is the path of a fixed point on the circumference of the circle A? In particular, what happens if the radius of A is exactly half the radius of B and A rolls around on the inside of B? Can you think of any use for this device?

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THE ALGEBRA LESSON

Stand firm in your refusal to remain conscious during algebra. In real life, I assure you, there is no such thing as algebra. —*Fran Lebowitz*