THE 1978 SUMMER SCIENCE SCHOOL

Last year saw the fourth University of New South Wales Summer Science School: 136 students from schools across New South Wales took part in projects in Mathematics, Physics, Chemistry and Optometry. Each project involved a week's work at the University in late November, doing research under the supervision of a member of staff. There were three mathematics projects:

"Models for Traffic Flow" with Karen Patterson (St. Scholastica's College), Douglas Niven (Homebush Boys' High) and Raymond Yu (Christian Brothers' College, Sutherland), supervised by Lyn Freeman;

"Some Applications of Euler's Theorem" with Craig Barrat (Barker College), Melanie Boileau (North Sydney Girls' High), Peter Dalton (Barker College), Mathew Dyer (Hurstvilie Boys' High) and Roland Luthier (Sydney Boys' High), supervised by Veronica Paul; and

"Statistical Models for Predicting University Marks from H.S.C. Marks" with Michael Drinkwater (Cranbrook), Scott Driver (The King's School), Anne-Louise Logan (Kincoppal Convent) and Nigel Lovell (Grafton High School), supervised by Dr. M.K. Vagholkar.

Here is a report on some of the activities of these groups.

BINARY MAGIC

V. Paul*

In the great temple of Benares, beneath the dome which marks the centre of the world, rests a brass plate in which are fixed three diamond needles, each a cubit high and as thick as the body of a bee. On one of these needles, at the creation, God placed sixty-four discs of pure gold, the largest disc resting on the brass plate, the others getting smaller and smaller up to the top one. This is the tower of Brahma. Day and night unceasingly the priests transfer the discs from one diamond needle to another according to the fixed and immutable laws of Brahma which require that the priest on duty must not move more than one disc at a time and that he must place this disc on a needle so that there is no smaller disc below it. When the sixty-four discs shall have been

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thus transferred from the needle on which at the creation God placed them to one of the other needles, tower, temple and Brahmins alike will crumble into dust, and with a thunderclap the world will vanish.

This is rumoured to be the origin of the Tower of Hanoi puzzle invented in 1883 by the French mathematician Edouard Lucas. (See Figure 1.) The problem is to transfer the eight discs to either of the two vacant pegs in the fewest possible moves, moving one disc at a time and never placing a disc on top of a smaller one. It is not hard to show that there is a solution regardless of how many discs there are in the tower. Can you discover a formula for the least number of moves needed to shift a tower of n discs? Can you prove it?

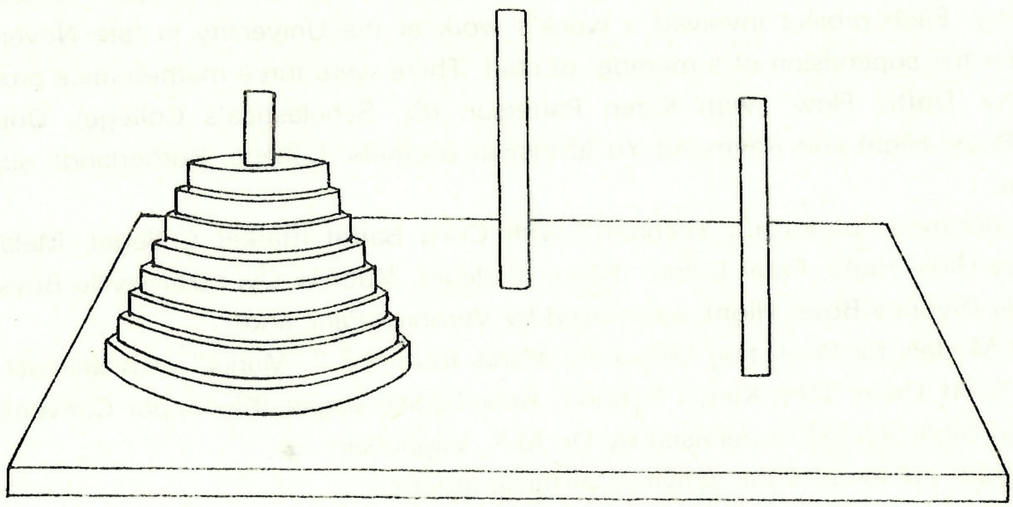

Figure 1

Consider a Tower of Hanoi with three discs; we label the discs, from top to bottom, X, Y and Z. We can shift the tower in seven moves as follows: At every second move, we shift the smallest disc X round by one peg anticlockwise. On the other moves, we make the only legal move which does not involve the smallest disc. (This is the shortest solution.) This procedure means that we move the discs in the following order: XYXZXYX. Try it and see.

Another popular nineteenth century puzzle was the Icosian Game invented in 1850 by the Irish mathematician William Hamilton. This game is played on a polyhedron as follows: Start at any corner of the solid and, by travelling along the edges, make a complete trip around the polyhedron, visiting each vertex once and only once, and return to the starting corner. You may enjoy tackling the round trip problem on some of the polyhedra described in the articles in Parabola, Volume 14, Numbers 1 and 2. Is it possible to find a Hamiltonian circuit of the sort just described on all these polyhedra?

Let us consider Hamiltonian circuits on the cube. We label the sides of the cube which are parallel to the X-axis with an X and similarly we label the other sides Y or Z. (See Figure 2.) If we trace a path along the edges of the cube, choosing edges in the order XYXZXYX, the path is a Hamiltonian circuit. Does this look familiar? Yes, it is the same as our solution to the Tower of

Hanoi with three discs! Can you extend this little surprise? Here is a hint: the order of transferring n discs in the Tower of Hanoi puzzle corresponds exactly to the order of edges in tracing a Hamiltonian circuit on the n-dimensional cube.

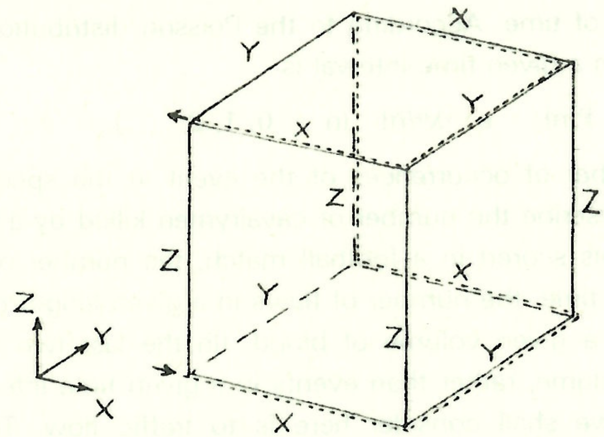

Figure 2

If you can find an old-fashioned ruler marked in eighths of an inch, you will discover another XYXZXYX. (See Figure 3.) Alternatively, we can generate this pattern in the following way. Write down the binary numbers from 1 to 7 and label the columns X, Y and Z, as in Figure 4. Now label each row with the letter which identifies the 1 that is furthest to the right in the binary representation. The sequence of letters is XYXZXYX again. Can you discover how to adapt this to solve the Tower of Hanoi with n discs? Can you find any more occurrences of this binary phenomenon?

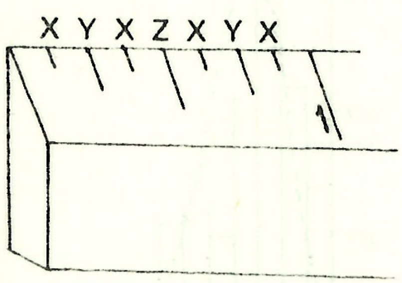

Figure 3

	Z	Y	X	
1	0	0	1	X
2	0	1	0	Y
3	0	1	1	X
4	1	0	0	Z
5	1	0	1	X
6	1	1	0	Y
7	1	1	1	X

Figure 4