TRAFFIC JAM

L. Freeman*

The Poisson distribution often gives a useful statistical model to describe the occurrence of isolated events in an interval of time. According to the Poisson distribution, the probability that a certain event occurs n times in a given time interval is

$$P(n) = e^{-\lambda}\lambda^n/n! \quad (n = 0, 1, 2, \ldots), \tag{1}$$

where λ is the average number of occurrences of the event in the specified time interval. This distribution can be used to describe the number of cavalrymen killed by a horse-kick in the course of a year, the number of goals scored in a football match, the number of decays occurring in a radio-active sample in a given time, the number of flaws in a given length of electrical cable, or the number of red corpuscles in a given volume of blood. (In the last two cases, we are counting events in a given length or volume, rather than events in a given time interval, but the principle is the same.) The application we shall consider here is to traffic flow. The Poisson distribution describes well uninterrupted, free-flowing traffic with cars travelling independently of one another, provided the average number of cars passing a given point in a specified time interval is less than 3.

Two experiments were performed to test the rather Utopian theory described above. The location for the first experiment was the main entrance to the University of New South Wales on Anzac Parade at 5.30 p.m. on a Monday afternoon. This location is at a bus stop and immediately after a set of traffic lights and the time was in the peak hour. Thus all the conditions for free-flowing traffic and a small average number of cars were violated. We did not expect the data to fit a Poisson distribution. The second location was near the overpass to the Airport Freeway at 2.30 p.m. on the same day. Traffic appeared to be flowing freely and the time interval of 10 seconds was chosen so that the average number of cars passing in one time interval was kept below 3. We

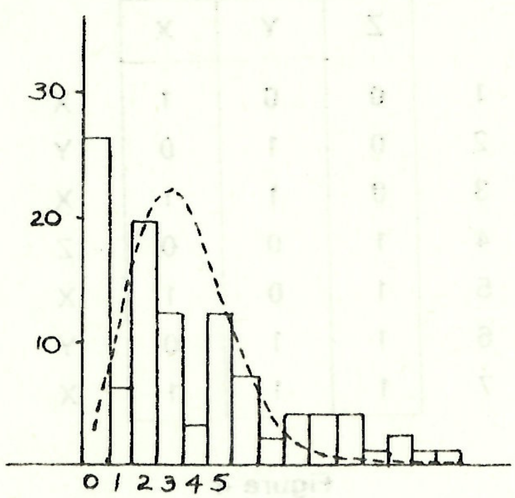

Figure 1

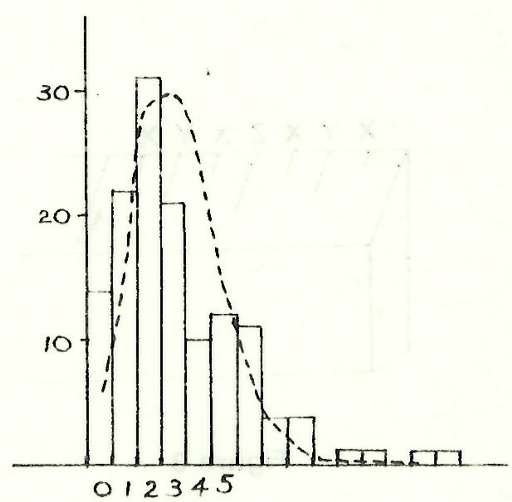

Figure 2

*Lyn Freeman is a Senior Tutor in the Department of Statistics at the University of New South Wales.

expected to find that the Poisson distribution described this data. (For the purpose of the experiment, cars were counted travelling in one direction only and "cars" included buses, trucks, etc. but not motor bikes.)

The results of the two experiments are shown in the histograms in Figures 1 and 2 respectively. For example, in the first experiment, no cars were recorded in 30 of the 10 second intervals, 1 car was recorded in 6 of the 10-second intervals, 2 cars were recorded in 19 of the 10-second intervals, 3 cars were recorded in 12 of the 10-second intervals, and so on.

In the first experiment with heavy interrupted traffic (Figure 1), the total number of 10-second intervals recorded is 103 and the average number of cars passing in a 10-second interval is $\lambda = 3.53$. With these values, we can calculate the expected number of 10-second intervals during which any given number of cars will pass from the formula (1). For example, the expected number of 10-second intervals with 2 cars recorded is

$$103\,P(2) = 103\,e^{-3.53}(3.53)^2/2! = 18.8.$$

These expected values are shown by the broken line in Figure 1. As we anticipated, the expected frequencies do not fit the observed results at all well. This emotional judgment can be confirmed by performing a chi-squared goodness of fit test and this does indeed indicate that the Poisson distribution could not be used to describe this particular type of traffic flow.

The same analysis was done with the data from the second experiment with freely flowing traffic, with the results shown in Figure 2. Again, a subjective judgment and a chi-squared test show that this data also cannot be described by a Poisson model. Apparently, the cars were not arriving independently as we had thought, but were moving in clusters. In fact, the location we used was at a section of the freeway which had traffic lights controlling the flow, but these were obscured from the experimental site by a sharp bend in the road. (This may also serve to confirm a widely held prejudice on the part of the great motoring public that there is no such thing as freely flowing traffic.) The experimental data can be described by the Neyman distribution, a more complicated affair, which copes with situations in which the quantity being observed arrives in clusters, but the various clusters arrive independently.

Can you find any freely flowing traffic?

ARE EXAMINATIONS ANY USE?

M. K. Vagholkar*

If examinations do nothing else, they at least produce vast quantities of numerical data. These numbers are used by all sorts of people, usually with the idea that they indicate a person's knowledge on a particular subject and that they can be used to predict the degree of success in employment or further study. For example, we might suspect that results in the Higher School Certificate examination could be used to predict performance in first year Mathematics at University. How can we test our suspicion? This sort of question is very common in statistics and it has led to many fruitful applications of statistics in practice.

**Dr Vagholkar is a Senior Lecturer in the Department of Statistics at the University of New South Wales.*