# PROBLEM SECTION

The first four problems below were set in the 1980 Canadian Mathematics Olympiad, and the next four in the 1979-80 Scottish Mathematical Challenge. Question 490 is quoted in Crux Mathematicorum, a Canadian problem solving journal, as an example of a problem proposed by a secondary school student: (Dougin Walker, Maryland).

Q. 479. If a679b is a five-digit number (in base 10) which is divisible by 72, determine a and b.

Q. 480. The numbers from 1 to 50 are printed on cards. The cards are shuffled and then laid out face up in 5 rows of 10 cards each. The cards in each row are rearranged to make them increase from left to right. The cards in each column are then rearranged to make them increase from top to bottom. In the final arrangement, do the cards in the rows still increase from left to right?

Q. 481. Among all triangles having (i) a fixed angle A and (ii) an inscribed circle of fixed radius r, determine which triangle has the least perimeter.

Q. 482. A gambling student tosses a fair coin and scores one point for each head that turns up and two points for each tail. Prove that the probability of the student scoring exactly n points is $\frac{1}{3}\{2 + (-\frac{1}{2})^n\}$.

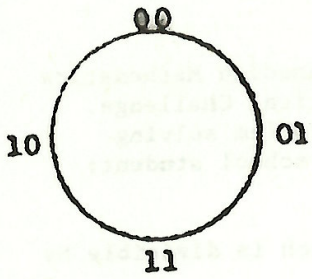
Q. 483. On Mathland T.V., a political commentator's summary of an election result was as follows:

"A Labour majority of 1729 last time has been turned into a Conservative majority of 1654 in this election, and the Conservative candidate has obtained 38% of the poll. Labour has taken second place. The Liberal has obtained only 14% of the poll and has been beaten into bottom place by the M.N.P. candidate, who has 50 more votes than the Liberal."

Given that there were just four candidates and that all the figures quoted were exact, determine the number of votes polled for each candidate.

Q. 484. Alan and Bill are serving at a hot drinks stall which provides only tea and coffee. The price of a cup of coffee is greater than that of a cup of tea, both prices being a whole number of pence. As each customer is served the amount paid is entered on a list. Alan sees that Bill has entered amounts of 30p and 61p on the list and knows that these are both incorrect as no transactions could give these totals. Alan himself has just served a customer with more than one drink and has, correctly, changed 19p. How much is a cup of coffee?

Q. 485.




There are four 2-digit binary numbers, namely 00, 01, 10 and 11. In the diagram, they are placed round the circumference of a circle in such a way that any two adjacent numbers differ in only one digit. Determine whether the eight 3-digit binary numbers can be placed round the circumference of a circle in such a way that any two adjacent numbers differ in only one digit. Is a similar arrangement possible for the sixteen 4-digit binary numbers?

Q. 486. A photograph shows four men, all the same height, standing in a straight line. The image of the first man is 3 cm high and the image of the fourth man is 2 cm high. One of the other two men was standing the same distance from the first man as from the fourth man: how high is his image? The remaining man's image is 2.5 cm high and he was 3 m from the first man. How far was he standing from the fourth man? Explain your working. (You may assume that all the images of the men are parallel to one another.)

Q. 487. Find all polynomials $P(x)$ satisfying the relation

$$P(2x) = P'(x) \cdot P''(x)$$

Q. 488. A and B lie on one side of a square of side length 1, and C lies on the opposite side. Let R be the radius of the circumcircle of $\Delta ABC$, and let $h < R < k$. What values of h and k make the relation true for all positions of the triangle?

Q. 489. A squad of soldiers are marching in Indian file at constant speed. The length of the file is 50 metres. A dog runs at constant speed from the last soldier to the first, then immediately turns round and returns to the last soldier. During this time the soldiers have moved forward 50 metres. How far has the dog run?

Q. 490. Show that $x^3 + y^4 = 19^{19}$ has no solution in integers $x, y$.