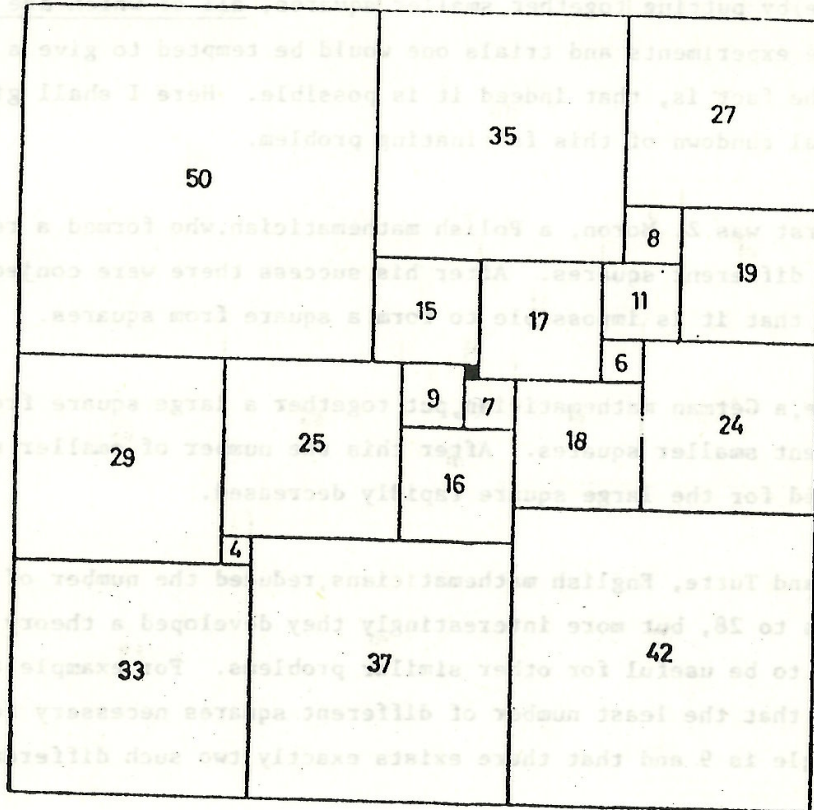# HOW MANY SQUARES FOR A SQUARE?

Let us pose the question: is it possible to form a rectangle (or even better a square) by putting together smaller squares, <u>all of which are different?</u>
After some experiments and trials one would be tempted to give a negative answer. But the fact is, that indeed it is possible. Here I shall give you a short historical rundown of this fascinating problem.

1925   The first was Z. Moron, a Polish mathematician, who formed a rectangle from 9 different squares. After his success there were conjectures about, that it is impossible to form a square from squares.

1939   Sprague, a German mathematician, put together a large square from 55 different smaller squares. After this the number of smaller squares required for the large square rapidly decreased.

1940   Stone and Tutte, English mathematicians, reduced the number of small squares to 28, but more interestingly they developed a theory which proved to be useful for other similar problems. For example they proved that the least number of different squares necessary for a rectangle is 9 and that there exists exactly two such different arrangements.

1948   Willcocks produced a square from 24 different smaller squares and this record was not broken for another thirty years.

1960   Bouwkamp, Dutch mathematician and his collaborators used the computer to investigate all the rectangles composed of 15 different squares, there are 3683 of them, but they found no squares among them.

1978   Duijvestijn, (also Dutch), used a DEC-10 type computer to show it is possible to form a large square from 21 different smaller squares, and he proved that this is indeed the smallest number.

The diagram of this last arrangement is shown on the next page.

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The numbers inside the squares represnet the length of a side in some units.
The tiny square blacked out in the middle has sides lengths 2 units

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┌─────────────┬──────────┬────────┐
│             │          │        │
│     50      │    35    │   27   │
│             │          │        │
│             │          ├────┬───┤
│             │          │  8 │   │
│             ├───┬──────┤    │19 │
│             │15 │  17  │ 11 │   │
│             │   │      ├────┴─┐ │
│             │   │      │  6   │ │
├──────┬──────┤ 9 │ 7 ├──┴──┬───┤ │
│      │      │   │   │     │24 │
│  29  │  25  ├───┴─┐ │ 18  │   │
│      │      │ 16  │ │     │   │
│      │      │     │ │     │   │
│      ├─┐────┴─────┤ │     │   │
│      │4│          │ │     │   │
├──────┴─┤          ├─┴─────┴───┤
│        │    37    │           │
│   33   │          │    42     │
│        │          │           │
└────────┴──────────┴───────────┘
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PATTERN MAKING WITH MATHEMATICS

Set up the table of addition of modulo arithemtic, with mod 3. Code each
element by a small pattern. Use this coded table to form a larger pattern by
rotating it thorugh 90° about its right hand lower corner.

+	1	2	0
1	2	0	1
2	0	1	2
0	1	2	0