CHANGE OF BASE

BY

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EDITOR'S NOTE: We received a letter from Catherine Playoust, aged 12 (just finished year seven) a student at Loreto Convent, Kirribilli. During her Christmas holiday she devised an algorithm to represent decimals in various bases. We are happy to print her work for your benefit and hope that it will inspire our readers for similar contributions:

NUMBER	BASE 2	3	4	5	6	7	8	9
.1	.00011	.0022	.012	.02	.03	.0462	.06314	.08
.2	.0011	.0121	.03	.1	.1	.1254	.1463	.17
.3	.01001	.0220	.103	.12	.14	.2046	.23146	.26
.4	.0110	.1012	.12	.2	.2	.2541	.3146	.35
.5	.1	.1	.2	.2	.3	.3	.4	.4
.6	.1001	.1210	.21	.3	.3	.4125	.4631	.53
.7	.10110	.2002	.230	.32	.41	.4620	.54631	.62
.8	.1100	.2101	.30	.4	.4	.5412	.6314	.71
.9	.11100	.2200	.321	.42	.52	.6204	.71463	.80

How to do:

Below is the method I used for working this table out. I shall use the example of Base Eight.

1. Write out the first eight exponents of the number of the base you are working with. (Write them in Base Ten.) These can be used for reference in the next step,

8, 64, 512, 4096, 32768, 262144, 2097152, 16777216.

2. Work out the value of $\frac{1}{10}$ in the base. Multiply the numerator and denominator of $\frac{1}{10}$ by the first exponent of the base which will make the numerator two digits. Then make the fraction the sum of two fractions, so that the first numerator is the highest possible number ending in zero below the main numerator. Then divide this first fraction by $\frac{10}{10}$, so the resulting fraction is a number over one of the exponents,

$$\frac{1}{10} \times \frac{64}{64} = \frac{64}{640} = \frac{60}{640} + \frac{4}{640}$$

$$= \boxed{\frac{6}{64}} + \frac{4}{640} .$$

Make out a table on which to put your information.

$\frac{1}{8}$	$\frac{1}{64}$	$\frac{1}{512}$	$\frac{1}{4096}$	$\frac{1}{32768}$	$\frac{1}{262144}$	$\frac{1}{2097152}$	$\frac{1}{16777216}$
. 0	6						

(Get your information from the circled fractions.)

Keep finding the value of $\frac{1}{10}$ in the same way, starting with the last fraction in the previous sum. It is easier not to simplify this fraction.

$$\frac{4}{640} \times \frac{8}{8} = \frac{32}{5120} = \frac{30}{5120} + \frac{2}{5120}$$

$$= \frac{3}{512} + \frac{2}{5120}$$

$$\frac{2}{5120} \times \frac{8}{8} = \frac{16}{40960} = \frac{10}{40960} + \frac{6}{40960}$$

$$= \frac{1}{4096} + \frac{6}{40960}$$

$$\frac{6}{40960} \times \frac{8}{8} = \frac{48}{327680} = \frac{40}{327680} + \frac{8}{327680}$$

$$= \frac{4}{32768} + \frac{8}{327680}$$

$$\frac{8}{327680} \times \frac{8}{8} = \frac{64}{2621440} = \frac{60}{2621440} + \frac{4}{2621440}$$

$$= \frac{6}{262144} + \frac{4}{2621440}$$

etc.

After each sum fill in your information (circled) on the table

$\frac{1}{8}$	$\frac{1}{64}$	$\frac{1}{512}$	$\frac{1}{4096}$	$\frac{1}{32768}$	$\frac{1}{262144}$	$\frac{1}{2097152}$	$\frac{1}{16777216}$
. 0	6	3	1	4	6	3	1

When the 'decimal' terminates or it becomes obvious that it recurs, you have the value of $\frac{1}{10}$ in the base in which you are working. $\frac{1}{10}$ in Base Eight is $.06314_8$.

3. Find the values of $\frac{2}{10}$, $\frac{3}{10}$, etc. by adding 'decimals' together. (Remember to add in the base you are working with.)

0.1+	.063146314...+		0.1+	.063146314...+
0.1	.063146314...		0.2	.146314631...
0.2	.146314630... = $.1463_8$		0.3	.231463145... = .23146

etc.

In adding one may notice that that the last written digit in the answer does not conform to the recurrence of the 'decimal'. This is because there seems to be no carrying from the column to the right, but there really is, since the 'decimal' goes on for ever.

Interesting things to notice:

In our decimal system, to multiply by 10 one moves the decimal point one place to the right. The same sort of thing can be done in other bases. For instance, in Base Two .00011 $(\frac{1}{10})$ after the decimal point is moved one place to the right it becomes .0011 $(\frac{2}{10})$. In Base Seven, .0462 $(\frac{1}{10})$ becomes .4620 $(\frac{7}{10})$. This effect can of course be noticed in all bases.

Base Five is very easy to do because $\frac{2}{10}, \frac{4}{10}, \frac{6}{10}, \frac{8}{10}$ are simply $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}$ and $\frac{4}{5}$ (.1, .2, .3, and .4 in Base Five). $\frac{3}{10}, \frac{5}{10}, \frac{7}{10}$ and $\frac{9}{10}$ are $\frac{1}{5} + \frac{1}{10}, \frac{2}{5} + \frac{1}{10}$ etc.

Base Nine is also very easy because the numbers follow a pattern where the first digit increases and the second digit lessens by one each time. However, one must remember that the two digits recur each time.

One of the Mathematics Projects during the Science Summer School last summer was "Let's Investigate Space". Part of the work concerned itself with filling the space (the plane) with equilateral polygons. The illustrations here and on pages 23 & 32 are some of the results using an equilateral 9-agon.