GEOMETRICAL TRANSFORMATIONS *

BY

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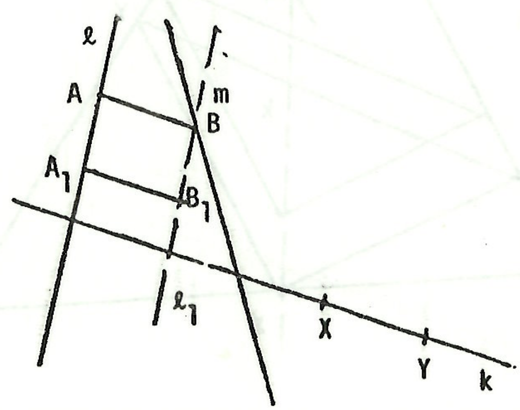
Transformations is a collective name for several different methods in Geometry. It means changing the configuration either by size or by position or both in such a way that some important aspects of it remain unchanged. We shall see that it can yield a powerful method in solving difficult problems or, in many instances, it can supply an "elegant" solution to a question. We start with a simple change in position.

Method I: Parallel translation.

We illustrate this method on a typical problem:

Problem 1: Given a straight line k, on which a segment $\overline{XY}$ has been marked; and also two other lines ℓ and m. Find the positions of point A on ℓ and B on m such that $\overline{AB}$ should be equal and parallel to $\overline{XY}$.

Solution: Let A_1 be any point on ℓ. Through A_1 draw a line parallel to k and find on this the point B_1 such that $\overline{A_1B_1}$ is = $\overline{XY}$. Now draw line ℓ_1 through B_1 so as to be parallel to ℓ. ℓ_1 intersects the line m in a point B and BA drawn parallel to k, A being a point on ℓ, will have the required length $\overline{AB}$ = $\overline{XY}$. (See Figure 1).



* This is the first part of an article by Mrs. E. Szekeres and indicates our intention to introduce more Geometry into Parabola, in the spirit of the new HSC syllabus.

As we see, the method here consisted by simply moving an unchanged figure (the line segment) parallel to itself. Try and see if you can apply a similar idea to solve the following problem:

<u>Problem 2</u>: Given two intersecting lines ℓ and m and a point P not lying on either line. Construct a straight line through P meeting ℓ in A and m in B such that B is the midpoint of AB.

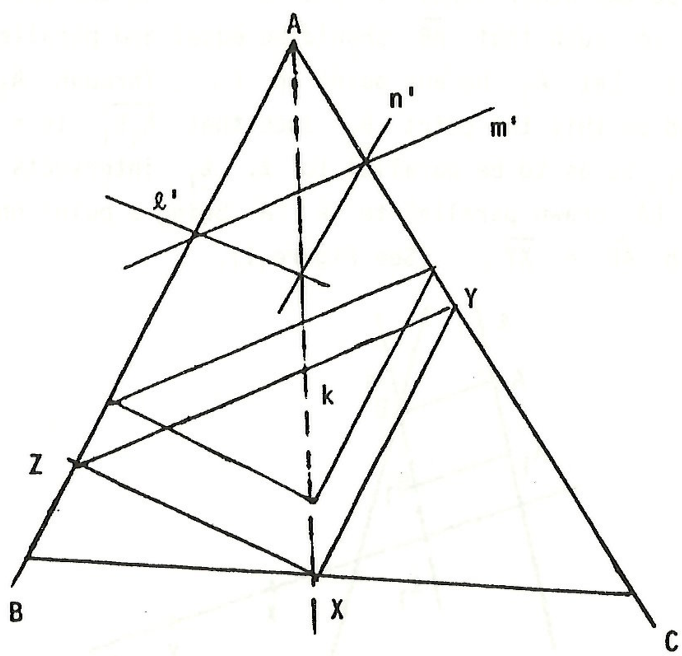
Sometime we combine parallel translation with changing the size but not the shape of the configuration. This is

<u>Method II</u>: <u>Similarity transformation</u>.

Here is a typical problem:

<u>Problem 3</u>: Given a triangle ABC and 3 pairwise intersecting lines ℓ,m,n. Construct a triangle XYZ such that its vertices lie on the 3 sides of triangle ABC and its sides should be parallel to the given directions ℓ,m,n.

<u>Solution</u>: All triangles whose sides are parallel to ℓ,m,n are similar to each other. Place one of these triangles so as to have one vertex on AB and another on AC. (See Fig.2).



If we allow this triangle to slide parallel with itself, always keeping its two vertices on AB, resp. AC, then its third vertex describes a straight line k

passing through A. This fact can be easily proved with the help of proportional segments. (Try it!). The line k meets BC in X and XZ, XY can be drawn in the required directions.

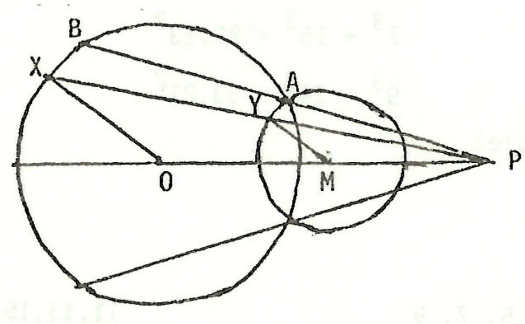
We see from this solution that the figure was moving parallel with itself changing in size, but always remaining similar to itself, hence the name similarity transformation. If two polygons are similar to each other and their corresponding sides are parallel, then, as in the problem above, the lines joining corresponding vertices will be concurrent in a point called the centre of similarity and the figures are centrally similar to each other. The method can be applied also to figures not bounded by straight lines, as the following example shows.

Problem 4. Given a circle, centre O, radius r, and a point P outside the circle. Construct a line ℓ through P meeting the circle in the points A and B such that PA = AB.

Solution: Let M be the midpoint of OP, and let an arbitrary line through P meet the circle O in point X. Let Y be the midpoint of PX. Then

$$\Delta XOP \sim \Delta YMP,$$
$$\therefore \quad MY = \tfrac{1}{2} r .$$

If we let X run through circle O, Y will describe a circle about M as centre and with radius = $\frac{1}{2}r$. We may consider this circle as having been obtained from O by a similarity transformation, centre P. Circle M and circle O intersect in point A and PA extended meets the circle O in B. (As it is clear from the figure, there are two solutions.) (See Fig.3).



Try this problem:

Problem 5: Given a Δ ABC, construct a square XYZU such that side XY lies along BC, while vertex Z is on AC, vertex U on AB.

Solutions and some other transformations will appear in our next issue.