PROBLEM SECTION

Q. 527. On a sheet of paper we read the following 100 statements:-

1. On this sheet exactly one statement is false.
2. On this sheet exactly two statements are false.

. ....

. ....

100 On this sheet exactly 100 statements are false.

Which (if any) of these statements is true?

Q. 528. Tom and Jim start at the same point of a race circuit and run in opposite directions round the track until Tom completes n laps, where n is an integer. Assume that both runners move at a constant speed and that Tom runs 3 laps in the time it takes Jim to complete 5. Express in terms of n the number of times the runners have met.

Q. 529. Colour each point of the plane with one of three colours. Prove that there is a segment of length 1 whose endpoints have the same colour.

Q. 530. We colour each point of a unit square (including the boundary) with one of three colours. Prove that there always will be 2 points of the same colour which have a distance at least $\sqrt{65}/8$.

Q. 531. Five married couples meet at a party and a number of handshakes take place. No one shakes hands with his wife and no one shakes hands twice with the same person. Mr. Smith asks each of the other people present how many handshakes he (or she) has made and discovers that no two of the 9 answers are the same. How many handshakes did Mr. Smith himself make?

Q. 532. The numbers $\{1, 2, 3, 4, \ldots, 1982\}$ are written on a blackboard. Any 2 of the numbers are chosen. They are erased, but their difference is written on the board. This operation is repeated over and over again until eventually there remains only one number on the board. Prove that it is an odd number.

Q. 533. The list of numbers 1 9 8 2 0 1 8 3 8 4 6 7 5 1 1 8 is constructed as follows:- The first pair of digits is 19, the next pair is 82. Adding 19 and 82 gives 101, of which the last two digits are taken as the third pair. The next pair is obtained by taking the last two digits of 82 + 01; and so on. Show that no four consecutive digits in this list spell out 1983.

Q. 534. The inhabitants of a strange planet use only the letters A and O. To avoid confusion, any two words which consists of the same number of letters differ from each other in at least 3 places. Prove that the language contains at most $2^n/(n+1)$ words with n letters.

Q. 535. We want to construct letters from the two Morse symbols (dots and dashes) such that (i) each letter consists of a string of 7 symbols, and (ii) if one of the seven symbols in a letter was transmitted erroneously, the letter should still be recognizable. (i.e. the string is more like the correct string than any of the other strings used for letters.) How many letters can this alphabet contain?

Q. 536. A number of circles are drawn in a plane in such a manner that there are exactly 12 points in the plane that lie on more than one circle. What is the smallest number of areas into which the plane can be subdivided by the circles?

Q. 537. A and B are points lying on opposite sides of a given line ℓ. How can one construct the circle passing through A and B having the shortest possible chord on the line ℓ?

Q. 538. If a,b are given non-zero numbers, find all values of x such that

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{x} = \frac{1}{a+b+x}.$$