PROBLEM SECTION

Q.551. Solve the system of equations

$$u^2 + v^2 + w^2 = 49$$

$$\frac{1}{u} + \frac{1}{v} + \frac{1}{w} = 0$$

$$u - v + w = 1$$

Q.552. Prove that if x, y and z are all positive numbers then

(a) $xy(x + y) + yz(y + z) + zx(z + x) \geqslant 6xyz$

and (b) $x^3 + y^3 + z^3 \geqslant 3xyz$.

Q.553. In a hat are six cards identical except for their colouring. Each side of every card is red, or white, or blue and no two cards are identically coloured. (So the cards are coloured RR, RW, RB, WW, WB, and BB). One card is drawn from the hat and placed on a table. The visible face is red. Find the probability that the hidden face is also red.

Q.554. Find three infinite sets A, B, C of non negative integers such that every non-negative integer can be written uniquely as the sum of an element from A, an element from B, and an element from C.

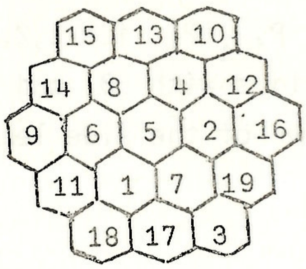
Q.555. The ratio of the speeds of two trains is equal to the ratio of the times they take to pass each other going in the same or in opposite directions on parallel tracks. Find that ratio.

Q.556. The points of (3-dimensional) space are coloured with one of five colours, each colour being used for at least one point. Prove that in some plane at least 4 different colours occur.

Q.557. A triangle is given. Assuming it is possible, show how to construct with ruler and compass a second triangle whose three altitudes are equal in length to the sides of the given triangle

Q.558. Show that 3 is the only positive integer value of n which is such that $2n-1$ is a factor of $(3n^2 - 3n + 1)(3n^2 - 3n + 2)$.

Q.559. If the numbers 1,2,3,. . . . . , N are placed in N small hexagons fitted together in a hexagonal pattern in such a way that sums along rows in any of the three directions through each small hexagon always come to the same total, the resulting arrangement is called a "magic hexagon".



The diagram shows a magic hexagon of order 3 (there are three small hexagons along a side of the hexagonal pattern). Totals along rows in any direction are all equal to 38.

(a) Show that there is no magic hexagon of order 2.

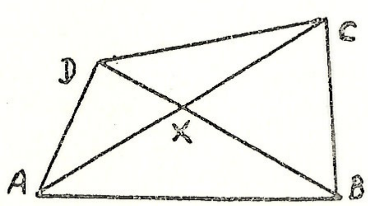
(b) Find N (the number of small hexagons) in the hexagonal pattern of order n, and find also the total along rows in a magic hexagon of order n.

(c) Show that if $n > 3$ there is no magic hexagon of order n.

(The result of Q.558 may be helpful).

Q.560. For an integer $n > 1$, let $f(n)$ be the product of all the positive divisors of n other than n itself. Find a simple description of all numbers n such that $f(n) = n$.

Q.561.

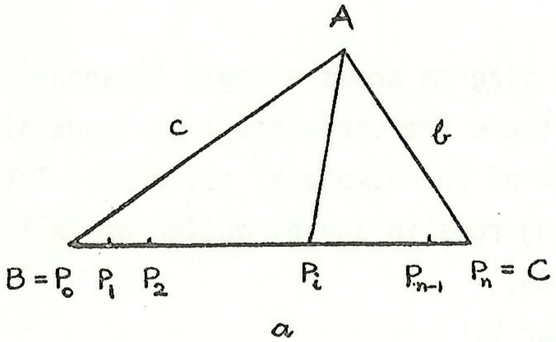


The figure shows a convex quadrilateral with both diagonals. There are eight triangles whose sides are sides of the quadrilateral, or diagonals, or segments of diagonals (viz ABC, BCD, CDA, ABD, ABX, BCX, CDX and ADX).

If all five diagonals are drawn in a convex pentagon there are 35 such triangles.

If no three diagonals of a convex n-gon are concurrent, find a formula for the number of triangles whose sides are all either sides of the n-gon, or diagonals of the n-gon, or segments of the diagonals.

Q.562. (Submitted by Howard Seo.)



Divide the side BC in ΔABC into n equal intervals $P_{i-1}P_i$, $i = 1,2,\ldots, n$ where P_0 coincides with B and P_n with C. Find (in terms of the side lengths a, b, c).

$$\lim_{n\to\infty} \frac{1}{n}(AP_1^{\,2} + AP_2^{\,2} + \ldots\ldots + AP_n^{\,2}).$$

Readers are warmly invited to submit solutions to any of the problems in this section. Please make sure that your solutions bear your name, year and school. Solutions to the preceding problems, together with the names of successful solvers, will be published in the issue after next.

To spur on your efforts, PARABOLA will award prizes for the best and most consistent solvers this year. Happy puzzling.

Solutions for the secret messages on page 7.

(1) Apparently the latter years of Elvis Presley were rather grizzly.

(2) If, to Man, the cricket seems to hear with its legs, it is possible that to the cricket Man seems to walk on his ears.

(3) If Chickens drank Coke, their eggs would not crack so often.