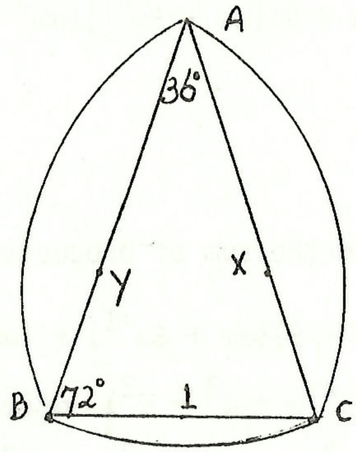
PROBLEMS



Q. 563. In the figure, X and Y are the centres of the circular arcs AB and AC respectively, and A is the centre of the circular arc BC. The triangle ABC has the following dimensions:- $\angle A = 36°$, $\angle B = 72°$, length BC = 1. Find the area enclosed by the three circular arcs.

Q. 564. In a triangle PQR, length PQ = length PR = s cms and length QR = 2b cms. Find the distance between the centroid and the circumcentre of the triangle.

Q. 565. A rectangular piece of paper is folded to place one corner on the diagonally opposite corner. The length of the fold is 65 cms, and of the long side of the rectangle is 144 cms. Find the length of the short side of the rectangle.

Q. 566. All the roots of the polynomial

$$x^5 - 10x^4 + ax^3 + bx^2 + cx - 32$$

are real and positive. Find a, b, and c.

Q. 567. Show that every whole number, N, has some multiple which contains only the digits 0 and 9. e.g. if N = 571428, then $175 \times N = 99999900$.

Q. 568. Without using calculus find the minimum value of

$$\frac{2 + 18x^4}{x^2}.$$

Q. 569. This is a long multiplication to calculate the square of a 4-digit number. All except the zero digits in the working have been obliterated, but the second last digit (i.e. the "tens" digit) of the answer is known to be odd. Find all the missing digits.

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        X X X X
        X X X X
      X X X X X
    X X X X X
    X X X X
  X X X X
X O O X O X X X
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Q. 570. "If two of my children are selected at random, likely as not they will be of the same sex", said the Sultan to the Caliph.
"What are the chances that both will be girls?" asked the Caliph.
"Equal to the chance that one child selected at random will be a boy" replied the Sultan.
How many children had the sultan?

Q. 571. *(A puzzle which appeared in a Canadian mathematical journal, CRUX MATHEMATICORUM, December 1982.)*

The following quiz could be plenty of fun. Just remember to work in base 21.

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NOW
 WE
ARE
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SIX ... A summation of fact;
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That ONE divides SIX requires no tact;
That TWO is prime is perfectly valid;
But SIX must be perfect, and thus ends our ballad.

[i.e., replace the capital letters by whole numbers in the range 0, 1, 2, ..., 20 so that working in base 21, the addition sum is correct, and the other conditions are also satisfied. A perfect number is equal to the sum of its factors; in the usual decimal notation the first few are 6, 28, 496, 8128, 33550336.]

Q. 572. A set consisting of integer numbers contains positive and negative numbers as well. If x and y are elements of the set then so are $2x$ and $x + y$. Prove that if any two elements of the set are chosen, their difference is also an element of the set.

Q. 573. A convex polygon is dissected into triangles by non-intersecting diagonals. Every vertex of the polygon is a vertex of an odd number of such triangles. Prove that the number of edges of the polygon is a multiple of 3.

Q. 574. Prove that if every element, starting from the second one, of an infinite list of natural numbers is equal to the harmonic mean of its neighbours, then all the elements of the sequence are equal.

[The harmonic mean of x and y is $\frac{2xy}{x + y}$.]

Problem 570 appeared recently in a newsletter of Melbourne University Mathematics Society; and several of the earlier problems (565 - 569) owe their inspiration to the same source. Problems 572-4 were used in the famous Hungarian Eotvos Competition many years ago.

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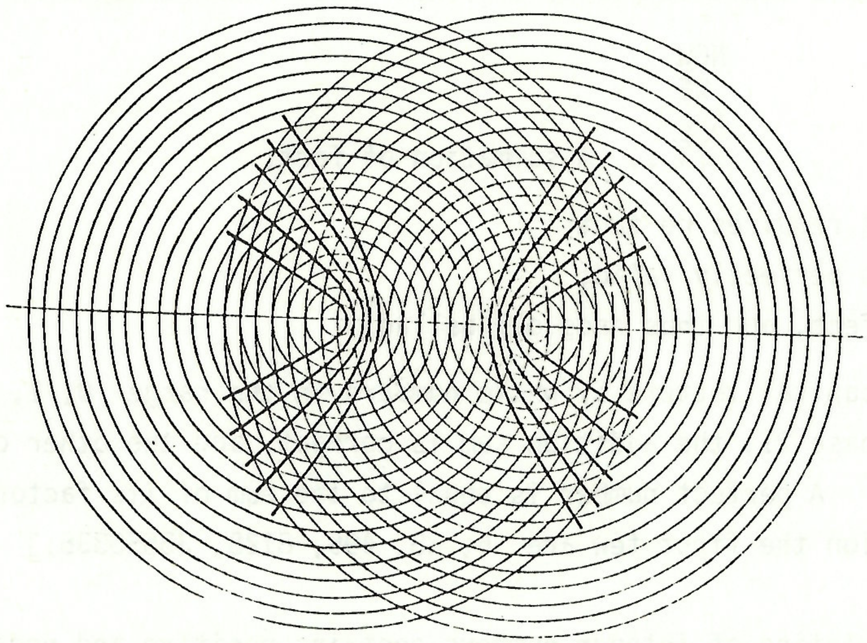

This is the answer to the question asked in our last issue on page 40. The family curves are hyperbolas.