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SCIENCE BEFORE THE CALCULUS

CREATIVITY OF GALILEO: THE LAW OF FREE FALL

BY

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There is much interest in the origins of science, with a view to understanding the creative processes of the human mind. In the 19th and early 20th century, when the scientific world view was in its heyday, it was commonly thought that the origins of modern science belonged to Galileo. Thus, in Oliver Lodge's *Pioneers of Science*, 1983 we read:

> "Galileo was not content to be pooh-poohed and snubbed. He knew he was right, and he was determined to make everyone see the facts as he saw them. So, one morning, before the assembled University, he ascended the famous leaning tower, taking with him a 100lb shot and a 1lb shot. He balanced them on the edge of the tower, and let them drop together. Together they fell and together they struck the ground. The simultaneous clang of these weights sounded the death knell of the old system, and heralded the birth of the new".

The trouble with this story is that it is a complete fabrication. As Galileo's Aristotelian opponents knew, in the everyday world, heavy bodies do fall faster than light ones. It is highly probable that the Tower of Pisa experiment was performed by one of Galileo's opponents and that Aristotle was proved right. In fact, in his *De Motu*, written while he was in Pisa, Galileo mentions objects dropped from high towers but he reaches the erroneous conclusion that the velocity is determined by the specific gravity of the body and that, hence, a ball of iron will fall faster than a ball of wool of the same weight. As Thomas Kuhn has pointed out in his book *The Copernican Revolution:* "Galileo's law (of free fall) is more useful to science than Aristotle's, not because it represents experience more perfectly, but because it goes beyond the superficial regularity disclosed by the senses to a more essential, but hidden, aspect of motion".

Galileo did not get his law from any new observation, but by thinking creatively and logically in the context of work presented to him by his predecessors.

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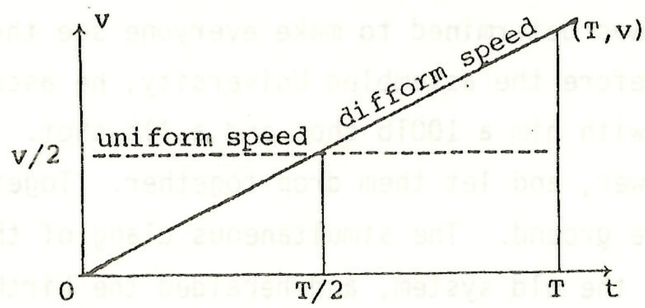
Predecessors dealing in Galileo's material can be found as early as the thirteenth and fourteenth centuries, and William Wallace has linked Galileo with them through the mathematicians at the College of Rome. Leonardo da Vinci should also be mentioned.

Mention of Aristotle is a reminder that great discoveries of a scientific kind occurred in every ancient culture, Babylon, Egypt, Ancient Greece, India, China, Islam etc., but in each case science suffered a stillbirth. Discoveries belonging to these cultures are in fact rediscoveries from another age, another culture. Reasons for this are worth contemplating. Could science die in our culture?

Movement in the 14th Century

Early in the fourteenth century, a group of mathematicians emerged at Merton College, Oxford, notably Thomas Bradwardine and Richard Swineshead. They developed the Merton rule, viz. "With respect to the space transcribed in a given time the latitude of motion uniformly difform is equivalent to its mean degree".

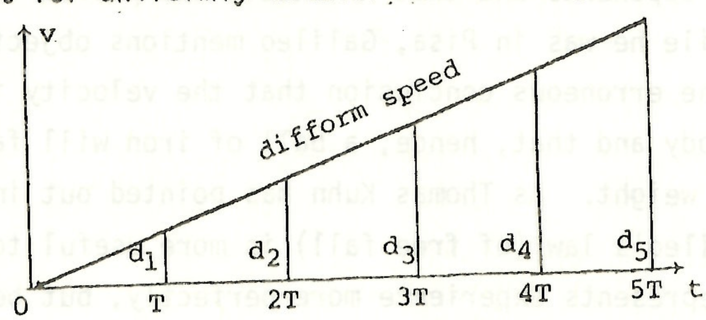
This can be easily understood using a velocity-time graph.



Merton Rule

For difform speed (uniformly accelerated motion) distance $d = \frac{1}{2}Tv$. For uniform speed at mean degree v/2, $d = T\frac{v}{2}$. (Note: In each case d is the area under the curve).

By 1350, the great French mathematicians, Jean Buridan and Nicholas Oresme had proved geometrically that for uniformly accelerated motion,



Oresme's Rule

the distances d_1, d_2, d_3, ... covered in equal intervals of time were in the ratio of the odd numbers, i.e. $d_1 : d_2 : d_3 : d_4 \ldots = 1 : 3 : 5 : 7 \ldots$

They apparently did not discover the result which Galileo developed, viz.

$$d_1 : d_1 + d_2 : d_1 + d_2 + d_3 : d_1 + d_2 + d_3 + d_4 = 1 : 4 : 9 : 16 \ldots$$
$$= t_1^2 : t_2^2 : t_3^2 : t_4^2 \ldots$$

or, as we say today, $\frac{S_1}{S_2} = \frac{t_1^{\,2}}{t_2^{\,2}}$.

Galileo used the medieval scholars precise definition of difform or uniformly accelerated motion as that motion, which beginning from rest acquires "equal moments of swiftness in equal times". Yet, for a long time Galileo mistakenly assumed that for a body under free fall,

$$\frac{v_1}{v_2} = \frac{S_1}{S_2} \; ;$$

i.e. velocity proportional to distance. Curiously however, in a letter to his friend Sarpi in 1604, Galileo gave the correct $S \propto vt^2$ law, accompanied by a false proof based on speed proportional to distance. It seems he reached the right conclusion for the wrong reasons. Even in his last book, where he got it all right, Galileo admitted that for a long time he believed it made no difference whether speed acquired was proportional to time or proportional to distance. By 1638 however, when his *Discourses on Two New Sciences* was published he wrote:

> "... I declare that I wish to examine the essentials of the motion of a body that leaves from rest and goes with a speed always increasing.. uniformly with the growth of time... I prove that the spaces passed by such a body to be in the squared ratio of the times... . ...I have been lucky that (the natural motion of falling heavy bodies) and the properties thereof correspond point by point to the properties demonstrated by me".

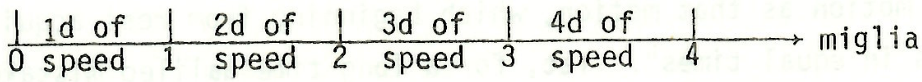
For Galileo to say he had been *lucky* was to put the situation mildly. By modern standards Galileo was talking nonsense when he sought a logical foundation for the law of free fall. He said that nature had *led him almost by the hand* to the discovery of the rule that equal speeds are added in equal times. I believe that this notion of "nature leading by the hand" to be closer to the truth than Galileo's historians would admit. We have only to look at the many non-sequiturs in Galileo's thought pattern to realise that something other than rationality and logic was at work. The creative process seems always to involve such non-rational elements. In Galileo's case the following elements can be discerned:

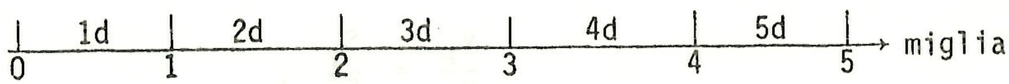
<u>(a) The Non-fatal Error</u>

Early ways of thinking about free fall acceleration were in terms of (what we would call today) quantum jumps of uniform speed. Galileo had the concept of "overall speed" from rest by adding up separate "degrees of speed". Thus

1 + 2 + 3 + 4 = 10 degrees of speed was the overall speed for four units of time; 1 + 2 + 3 + 4 + 5 = 15 degrees of speed for five units ot time. His basic task was to

seek the rule of *proportionality* for the uniform growth of distances, times and speeds in free fall. He first tried (as any good Platonist would do) thinking logically about arbitrary examples.

(1) 

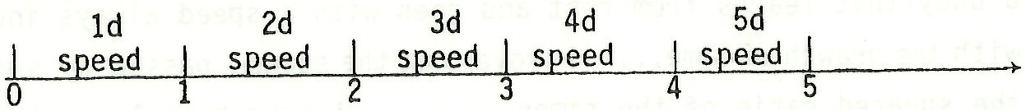
(2) 

(1) 4 miles with 10 degrees of speed in 4 hours.

(2) 9 miles with 15 degrees of speed in 5? hours.

To get up to 4 degrees of speed takes four units of time, and his rule was: one more degree of speed for each additional mile. Consistently with his speed-time concept he wrote:

(1) 4 miles with 10 degrees of speed in 4 hours. He next meant to add 5 degrees of speed for the next mile



(i.e. 5 miles with 15 degrees of speed in 5? hours)

but what he actually wrote was:

(2) 9 miles with 15 degrees of speed in 5? hours. Basically, he was trying to establish a consistent usage for his notion of "degrees of speed". The error of writing 9 instead of 5 was crucial in subsequent developments. The 5? meant that the time was to be examined. We next note:

(b) Some Happy Co-incidences

Galileo was basically a mathematical physicist. He expected physics to conform to elementary mathematical notions like Euclidean proportion. To get a ratio of distances and times Galileo substituted 6 hours for the four hours in Statement 1.

(1a) 4 miles with 10 degrees of speed in 6 hours. With this change, in order to cover 4 miles in 4 hours it is necessary to travel at 15 degrees of speed:

(3) 4 miles with 15 degrees of speed in 4 hours.

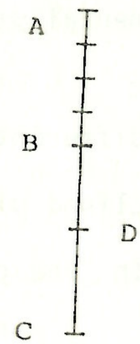
[Note: $\frac{10}{15} = \frac{2}{3}$; $\frac{6}{4} = \frac{3}{2}$]

Galileo noted that if "15 degrees of speed" is an overall rate then

(3a) 8 miles in 8 hours at 15 degrees of speed.

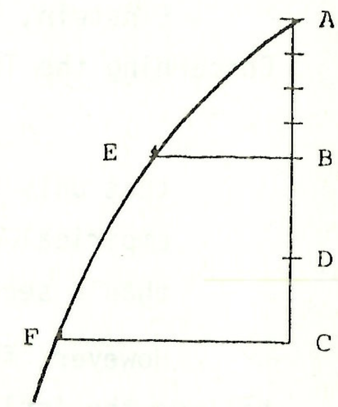
This was an immediate contradiction of statement (2). Had he put 5 instead of 9 in statement (2) he would have got consistency in times and distances but it would have contradicted the basic idea of uniform acceleration.

Galileo next drew a vertical line and lettered it A, B, C (AB = 4 units, AC = 9 units). He added D, so that AD was the mean proportional between AB and AC, (i.e. $\sqrt{4} \cdot \sqrt{9} = 2 \times 3 = 6.$). He used this to get the times correct. The time through the shorter distance 4 miles at 10 degrees of speed was 4 hours, but the time through the longer distance 9 miles at 15 degrees of spped was 6 hours, the mean proportional of the *distances* from rest. AB represented the time (and distance) in falling 4 units and AD the time in falling AC.



Galileo was indeed lucky to start with 4 and 9 and lucky with the numbers 10 and 15 because when two objects fall from rest through distances in the ration 4: 9, both their average *and* their overall speeds do have the ratios 10: 15 = 2: 3. Had he started with 9 and 16 this would not have happened.

Galileo next extended his diagram by drawing BE as shown, AB = BE and BE represents the speed acquired at B. When he calculated the placement of the point F by the ratio he had developed, [$\frac{BA}{AD} = \frac{BE}{CF}$] he found that F lay, not on AE but on a parabola through A and E.
[Galileo had got his doctorate and his professorial position at Pisa for his geometry of the parabola].



His argument goes like this:

$$\frac{AB}{AD} = \frac{AD}{AC} \qquad \text{[AD the mean proportional between distances AB, AC]}$$

Let $\frac{AB}{AD} = \frac{BE}{CF}$, where BE is speed at B, CF is speed at C.

Therefore $\frac{AD}{AC} = \frac{BE}{CF}$, so $\frac{(AD)^2}{(AC)^2} = \frac{(BE)^2}{(CF)^2}$ i.e. $\frac{AB \cdot AC}{(AC)^2} = \frac{(BE)^2}{(CF)^2}$

Therefore $\frac{(BE)^2}{(CF)^2} = \frac{AB}{AC}$ [i.e. $\frac{v_1^2}{v_2^2} = \frac{s_1}{s_2}$]. Thus E, F are on a parabola through A.

Now $\frac{v_1}{v_2} = \frac{t_1}{t_2}$ [This is the motion he is examining]. Therefore $\frac{S_1}{S_2} = \frac{t_1^2}{t_2^2}$.

There is very little logic in the whole argument except the "logic" of discovery.

(c) Empathic Involvement with Subject

What had to happen next was what Galileo called "sensate experience and necessary demonstration". He needed a postulate which would link free fall with a possible experimental procedure. He spoke of

> 'the only principle: equal speeds are acquired in descent along different inclined planes if the planes are of the same height'

The inclined plane would be vertical in the case of free fall.

In "The dialogue Concerning Two New Sciences", Salviati (who is Galileo) says:

> "For the present let us take it as a postulate, the *absolute truth* of which will be established when we see other conclusions built on this hypothesis correspond to and agree perfectly with experience".

This would seem to be an early instance of the hypothetico-deductive method in science. The only objection that could be made to-day would be his use of the words 'absolute truth'. It was this, in another context, which caused him to run foul of the Roman Holy Office. However, that is another story.

Einstein, in his Preface to Stillman Drake's translation of Galileo's, "Dialogue Concerning the Two Chief World Systems" says that

> "... the experimental methods at Galileo's disposal were so imperfect that only the boldest speculation could possibly bridge the gaps between the empirical data. For example there existed no means to measure times shorter than a second".

However, from evidence Stillman Drake has produced, Galileo's success with the ball-on-the-inclined-plane experiment was due to Galileo's involvement with music and his consequent ability to measure time with his own "internal' clock. It is recorded that Toscanini, for example, would never be a second out in his timing of a piece lasting 40 minutes. A musician can maintain a highly accurate beat according to an internal rythm and by concentration on inner areas of experience he can detect extremely small departures from evenness.

Galileo set up a shallow inclined plane, rising 60 points in 2,100 points of distance. He marked distances down the plane run through by a steel ball in equal time units of about half a second and marked the distances using a fret made of gut (as were used on musical instruments of the time) at the precise time intervals. Stillman Drake was able to reproduce Galileo's measurements very nearly, marking the distances with rubber bands and singing "Onward Christian Soldiers" at a rate of two beats per second. Galileo may have used a march tune. Galileo declared finally that the ratio of the distances agreed with the squared ratio of the distances agreed with the squared ratio of the times to "within a tenth of a pulse beat".

Inclined Plane Experiment

	0	1	2	3	4	5	6	7	8 Time
		33	130-	298+	526+	824	1192-	1620	2104 Distance
		33	132	297	528	825	1188	1617	2112 Theoretical Distance
i.e. 33x		1	4	9	16	25	36	49	64

1 : 3 = 1+3 : 5+7 = 1+3+5 : 7+9+11 = 1+3+5+7 : 9+11+13+15 etc.

The table above shows Galileo's figures for each of the equal units of time, the first fret being at 33 points down the plane. His plus and minus signs indicate his estimations of whether the times the ball crossed the fret were slow, fast or exact. The second line of figures are the exact values of the theoretical distances. Note that because of the properties of proportionality of the successive sums of the odd numbers, any unit of time could have been used. It seems that the actual value of Galileo's unit of time was 0.55 seconds.

This episode seems to me to illustrate the principle of creativity that a creative scientist or artist applies, namely to establish a connatural relationship with the subject matter. This means an empathic openness to the possibility of "letting nature speak". This is not a rational mode of consciousness.

(d) Contradictory Understandings

It was one thing to get the distance-time square rule correct but another to find, as Galileo says, an "indubitable principle" from which to prove it. In his letter to his friend Sarpi, Galileo gave the rule correctly but implied that the rule stemmed from the contradictory assumption that the speeds acquired are proportional to the distances from rest. He preferred this since it was an aesthetically pleasing result based on the Archimedean diagram for an arithmetic progression. He had an idea fixé about speed in free fall from observing that a pile driver doubles its effect by dropping a weight from a doubled height. The creative mind uses analogies but sometimes they can be deceptive. Until his last book Galileo had believed that it made no difference whether speed was proportional to time or distance. It took nature to 'lead him by the hand' to see the difference. Creativity can survive fixed misleading concepts, logical oversights and false analogies. In the case of Galileo it had also to survive sheer pig-headedness and over enthusiastic self-praise.

(e) New Philosophic View Point

Galileo, together with Copernicus and Kepler, was a representative of the new philosophy of his time, in marked contrast to the established philosophy of Aristotle. Viable science had begun two and a half centuries earlier from an Aristotelian context. There is a lesson here for us, since the new philosophies of today are in marked contrast to the empiricist and positivist philosophies of the scientific Enlightenment. What was new in Galileo was part of an older tradition stemming from Pythagoras and Plato. The reality behind the world of change was discoverable through the discernment of simple mathematical regularities in nature.

The table shows Galileo's measurements for the distances travelled in equal times by the ball on the inclined plane.

Note that the distances apart are proportional to

$$1 \quad 3 \quad 5 \quad 7 \quad 9 \quad 11 \quad 13 \quad \ldots$$ i.e., the gnomon of the square, and

$$1 : 3 = 1+3 : 5+7 = 1+3+5 : 7+9+11 \text{ etc.}$$

so that whatever unit of time was used the same ratios of distances to times would be found.

For the Platonist this progression of order from the One was something like a proof of Plato's supreme idea of Good. This was what the Christians called God. For the Platonist such ideas are more compelling than Galileo's "sensate experiences and necessary demonstrations".

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OLYMPIAD OMNIBUS (Continued from page 18)

Problem 0.0.3

Find all of the integer solutions of

$$\frac{1}{x} + \frac{1}{y} = \frac{2}{1983}.$$

You should state clearly the reasons for your answers. Solutions should be sent to the writer: G. Ball, Department of Mathematics, University of Sydney, NSW 2006.

Solution to this question will appear in the next Parabola along with other Olympiad Information.

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