EULER

BY

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It is a remarkable fact that quite ordinary families can suddenly throw up quite extraordinary mathematical genius. Isaac Newton was an example - and so was the much less well-known Leonhard Euler. So great was his talent and importance that in the field of mathematics the eighteenth century has been described as 'The Age of Euler'. The year, 1983, marked the bicentenary of Euler's death, and it is appropriate to introduce this truly great man to readers who may be hardly aware that he even existed.

Leonhard Euler was born on 15 April 1707, and spent his childhood in the village of Riehen, near Basle, Switzerland. His forefathers had been mainly craftsmen, but his father, Paul, had achieved an academic education and became a Protestant minister. During his studies, Paul Euler had attended mathematics lectures given by the eminent professor at Basle University, Jakob Bernoulli. Interestingly, the Bernoulli family show that mathematical genius is sometimes inherited - no fewer than seven of them were outstanding mathematicians!

Leonhard Euler's father taught him elementary mathematics, and he then studied by himself for some time before receiving guidance from a private tutor. At the tender age of 13 years, he entered Basle University to study for a 'general arts degree'. (Such an early age of entry was by no means as uncommon or surprising than as it appears to us today.) Being bored with his normal studies, he sought the additional challenge of mathematics in his spare time. Fortunately for the young Euler, Johann Bernoulli, who had succeeded his brother Jakob to the post of professor of mathematics at Basle in 1705, was on hand to give expert advice, and Euler was able to progress well with his private

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studies. In 1722, at the age of 15, Euler obtained his Batchelor of Arts degree, and in 1723 he followed this with his Master of Philosophy degree.

Complying with his father's wishes, he then joined the department of theology to study Hebrew, Greek and theology, but his heart was really elsewhere. Although he was to remain a committed Christian throughout his life, Euler wisely rejected the path to becoming a church minister, and turned to mathematical and scientific enlightenment. His investigations began in 1725, when he was just 18 years old.

Finding a job was always a big problem for an aspiring mathematician in those days. There were few universities, and those which did exist were small - Basle, for example had barely 100 students, and fewer than 20 'professors'. Having failed to secure a vacant post at Basle in 1727, probably because of his youth rather than through lack of merit, he took up a position in Russia at the newly formed St. Petersburg Academy. It was at about this time that his work on exponential functions led him to introduce the familiar symbol e for the important transcendental number 2.7128...

Although his new job really required him to study physiology, he was able to work in the mathematical field, in the company of eminent specialists in geometry, trigonometry, analysis, number theory, mechanics, astronomy, cartography, etc.

One function of the St. Petersburg Academy was the solving of surveying and other technological problems for the Russian government. This led Euler to apply his genius to a very wide range of problems - in map construction, shipbuilding, navigation, etc. Nevertheless, what to us would appear major scientific achievements were but minor entertaining interludes to Euler. By the year 1741, when he was 34 years old, he had published no fewer than 55 major mathematical papers, and had another 30 already written! It is doubtful whether any man before or since has had such a fertile mind. An indication of this is that he had so many brilliant ideas when a young man that sometimes it was only 20 years later that he had time to work out all the details for publication!

During this early period he discovered the amazing relationship, which bears his name, connecting e, π and i (his own invented symbol for $\sqrt{-1}$):

$$e^{i\pi} = -1,$$

and, he investigated the Euler constant, represented by

$$y = \lim_{k\to\infty} \left\{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{k} - \log_e k\right\} \simeq 0.5772.$$

This result can be very useful for estimating the sum of the finite series $\sum_{k=1}^{n} 1/k$, since his result showed that

$$1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} \simeq y + \log_e(n).$$

An application of this is in determining how many packets of chewing gum one must expect to buy in order to get a complete set of picture cards, if one is given free with each packet! It can be shown (you are invited to try) that the appropriate formula for n different cards in a set is:

$$\text{expected number} = n\left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}\right).$$

Thus, for $n = 25$, an exact calculation of the above gives 95.4 (rounded to 3 significant figures), and the use of Euler's approximation gives 94.9, a surprisingly accurate result, with *much* less effort.

The political atmosphere in Russia became turbulent, and in 1740 Euler was pleased to accept an invitation to move to Berlin (then in the state of Prussia), to help reorganize the decaying Berlin Society of Sciences. He went in 1741, taking with him his wife Katharina, whom he had married in 1733, and their two sons. Such was Euler's energy that, after his move, he continued to work for the St. Petersburg Academy whilst simultaneously building up the Berlin equivalent! Just as he increased the reputation and activities of the Berlin academy, so did he build up the size of his family, with the addition of a third son and two daughters. It is recorded that a further eight children died in early infancy. This reminder that not all was as rosy as we might

be led to believe is strengthened by noting that, as the result of a disease, Euler lost the sight of one eye in 1738, although it did not diminish his own vigour or output one jot.

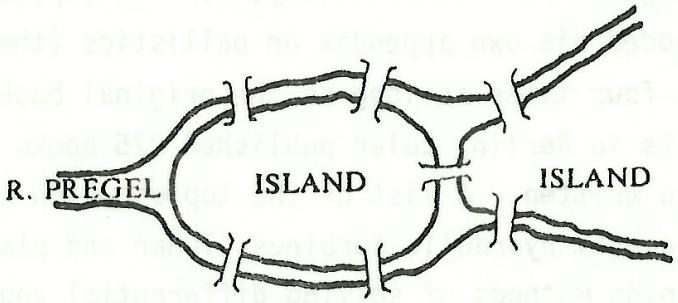


Figure 1.

Euler was the first to study the now major mathematical area of topology. One simple example of this is his solving of the problem of the seven bridges of Königsberg (Figure 1). The problem was to devise a route to enable one to walk over each bridge in turn without crossing any bridge twice. (The reader may like to investigate this.)

The general approach to this kind of traversability problem is to convert it (Figure 2) into a simple network involving nodes (i.e. points), arcs (i.e. lines) and regions (i.e. areas) (Figure 3). This reduces to a network with

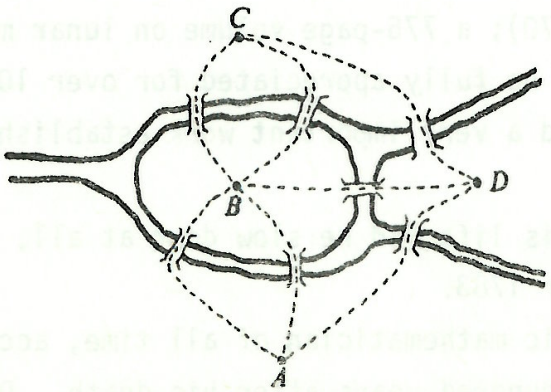


Figure 2.

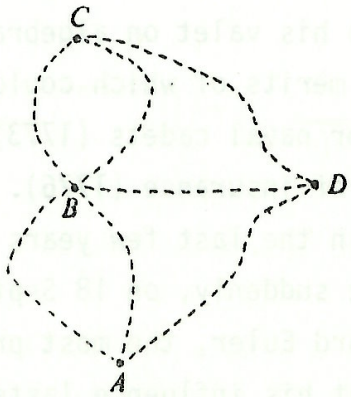


Figure 3.

one 5-node and three 3-nodes. Euler was able to show that any network with no more than two odd-nodes is traversable. (The reader may like to investigate this.)

Euler wrote mainly in Latin and French, but was conversant with other languages too. This is illustrated by the fact that he translated into German the important English work on artillery, *New Principles of Gunnery*, by B. Robins. Euler added his own appendix on ballistics (the theory of firing projectiles) which was four times as long as the original book itself!

During his 15 years in Berlin, Euler published 275 books and papers, with a further hundred or so written. A list of the topics which he studied includes: the theory of toothed gears, hydraulic turbines, lunar and planetary motion (which involved developing methods of solving differential equations, which bear his name), optics, magnetism, electricity, hydrodynamics, the theory of calculus, etc.

At the age of 59, when most would have been thinking about a cosy retirement, Euler left Berlin to return to the St. Petersburg Academy to continue his work there. Soon after his arrival in 1766, he suffered almost complete loss of sight in his remaining eye. He could see only dim outlines of large objects, and so was quite unable to read or write. Even this major setback could not quell the flow of ideas and publications. He produced almost as many books and papers after his blindness as he did before! He used his sons and students as collaborators, and his valet as an amanuensis. He continued to produce work of the highest standard - often following up brilliant ideas formulated years earlier. Some of his major works in this period were: three volumes on optics (1769-71); three volumes on integral calculus (1768-70); two volumes dictated to his valet on algebra (1770); a 775-page volume on lunar motion (1772) the merits of which could not be fully appreciated for over 100 years; a manual for naval cadets (1773); and a very important work establishing the principles of insurance (1776).

Only in the last few years of his life did he slow down at all, and he died, quite suddenly, on 18 September 1783.

Leonhard Euler, the most prolific mathematician of all time, accomplished so much that his influence lasted a hundred years after his death. During his lifetime he published about 560 substantial articles and books. It took eighty

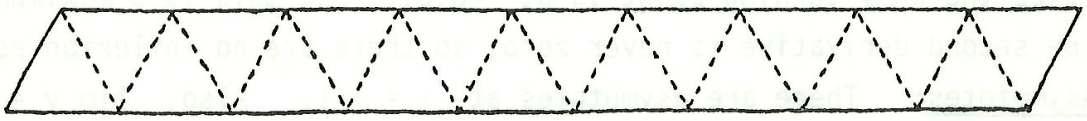
years after his death to edit and publish the memoires he left behind. Collection and publication of his complete works, or *Omnia Opera*, has only recently been completed, and has been one of the most important undertakings in the history of science. The *Omnia Opera* consist of 29 volumes on mathematics, 31 volumes on mechanics, and 12 volumes on physics and other topics. In all, there are 72 massive volumes of original work, currently available, should you wish to buy the set, for 9880DM (about £3000).

Reference:

An excellent short account of Euler's life and work may be found in Carl Boyer's *A History of Mathematics*, Chapter XXI. (Wiley, New York, 1968).

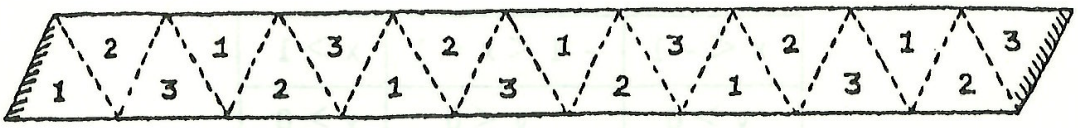
TO MAKE A HEXAFLAGON

Step 1.



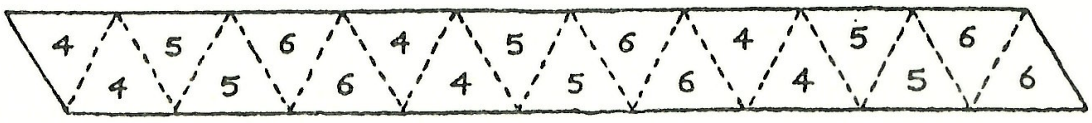
On a strip of thin cardboard draw and score 18 triangles.

Step 2.



Number triangles on one side, and mark the edges of end triangles as shown.

Step 3.



Turn the strip over and number triangles as shown.

(Continued on page 22.)