PROBLEM SECTION

You are invited to send in written solutions to any or all of the following problems. Solutions will appear in Vol.21 No.2.

Q.612. From a point P inside a cube, line segments are drawn to each of the eight vertices of the cube, forming six pyramids each having P as the apex and a face of the cube as the base. Is it possible to place P in such a position that the volumes of the pyramids are in the ratio $1:2:3:4:5:6$?

Q.613. Find all pairs of integers x,y such that

$$5x^2 + 5xy + 5y^2 = 7x + 14y.$$

Q.614. ΔOAB is rightangled at O, and P is a point in the plane of the triangle such that a rightangled triangle can be constructed with sides equal in length to OP, AP, and BP. Find the locus of P.

Q.615. Let f be a function satisfying the functional equation

$$f(s,t) = f(2s + 2t, 2t - 2s)$$

for all real numbers s and t. Define g by $g(x) = f(2x,0)$. Prove that $g(x)$ is a periodic function. [i.e. Show that for some real number p, $g(x + p) = g(x)$ for every x.]

Q.616.

1	2	3	4	5	-	-	-
2	4	6	8	10	-	-	-
3	6	9	12	15	-	-	-
4	8	12	16	20	-	-	-
5	10	15	20	25	-	-	-

In the above array the entries in the nth row are successive multiples of n. Find a formula for S_n, the sum of the entries in the nth diagonal. [e.g. S_5 = sum of the numbers in the box = 35.]

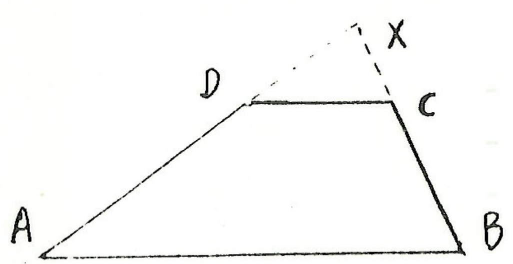
Q.617. Let n be a positive integer and s any real number greater than 1. Prove that

$$1/1^s + 1/2^s + 1/3^s + \ldots + 1/n^s < 1 + 1/(2^{s-1} - 1) .$$

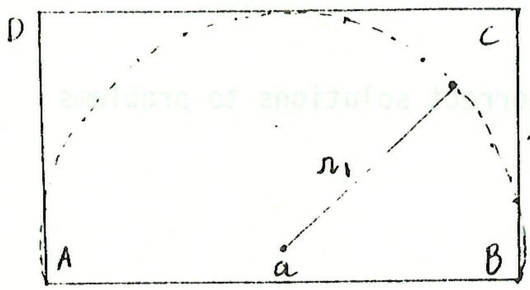
Q.618. Let t,u,v,w,x,y,z be non-negative numbers whose sum is 1, and let m be the maximum of $t + u + v$, $u + v + w$, $v + w + x$, $w + x + y$ and $x + y + z$. Find the smallest possible value of m, and show how to choose the numbers to achieve that value of m.

Q.619. Two pirates bury their treasure in a field using 2 tall trees, A and B as landmarks. Not too far from the trees they find a rock, C. One pirate paces out from C to B, turns 90° right, and paces out a distance BD equal to CB. The other pirate similarly walks from C to A, turns 90° left, and goes to E, making AE equal to CA. Then they walk towards each other, meeting at the point X half way between D and E, where they bury the treasure. When they return to recover the treasure the trees are easily recognized, but owing to a landslide there are now hundreds of rocks indistinguishable from C covering the field. Can they find the treasure?

Q.620. The base AB of the trapezium ABCD is fixed, but the parallel side CD is moved (remaining parallel to AB) so that neither its length nor the perimeter of the trapezium is changed. Find the locus of the intersection X of AD and BC.



Q.621. In the rectangle ABCD let length AB = a and length BC = b.



Let r_1 be the radius of the circle through A and B which touches CD, and let r_2 be the radius of the circle through A and D which touches BC.

Prove that $r_1 + r_2 \geq \frac{5}{8}(a + b)$.

Q.622. Let N be any perfect square. If its digits are added together, the number S(N) results. If this operation is repeated often enough, a one digit number is obtained. Show that it is always one of 1,4,7 or 9.

e.g. $3892 \times 3892 = 15147664$ and $S(S(15147664)) = 7$

$9262 \times 9262 = 85784644$ and $S(S(S(85784644))) = 1$.

Q.623. Andy, Bert, and Colin, having tied for first place in their chess club tournament, are to play off for the championship. Each is to play one game with both the others, scoring 1 for a win, ½ for a draw, and 0 for a loss. If their scores are still all level, this will be repeated. However if two of them are level ahead of the third, those two will continue to play until one of them scores a win.

Andy plays sound but cautious chess. He never loses against either of the others, but has probability $\frac{1}{10}$ of beating Bert in any given game, and probability $\frac{1}{5}$ of winning against Colin. Games between Bert and Colin are swashbuckling affairs that never result in draws; Bert wins 60% and Colin 40%. Compare Andy's and Colin's chances of emerging as club champion.

PROBLEM SOLVERS

Only two people sent in their solutions to the problems in Vol.20 No.1, but their solutions were both very good.

K. Boroczky (St. Patrick's College) sent correct solutions of problems 588, 589, 591, 592, 594, 595, 596 and 598(i). His solution of 593 was also correct if the machine is permitted to double an available number (not assumed in the solution published).

L-A Koe (James Ruse Agricultural H.S.) sent correct solutions to problems 589, 590, 591, 592, 597 and 599.

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Correction

In Problem 84.6 (on page 22 of the previous edition of Parabola) the expression $2\sin^2 A - 1$ should be $1 - 2\sin^2 A$ and the three solutions in the required range are 56°, 124°, 270°, correct to the nearest degree.

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