IT'S ONLY RELATIVE (ESPECIALLY)

BY

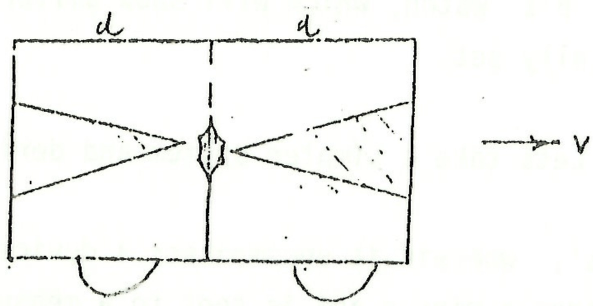
RONNIE BRAVERMAN*

If you ask anyone about the theory of relativity, there is a very high probability (somewhere extremely close to one I imagine) that they could tell you the following:

1. Albert Einstein developed it;
2. the speed of light, c, is independent of any object;
3. it is the most complicated theory (not many believe there is evidence to back it) in the history of mankind.

And this is very unfortunate. One, because the special theory of relativity is not beyond most of those who can understand Parabola, and secondly, some fantastically interesting results can be observed via the theory.

For example: imagine that a train is moving in a straight line motion, and within the exact centre of the train car there is a device which can simultaneously (used in a very loose way, as we shall see later) send a beam of light towards the front and back of the car. Now put yourself in the train, which is travelling at velocity v. Relative to you, however, the train is stationary and thus when the device sends out two beams, they will travel at speed c (all relative to you and the train don't forget), and thus will reach the two ends of the train at precisely the same time.



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Now imagine that you are sitting on a hill watching the train. When the device goes off, it will send out two beams of light at speed c (remember that light has a velocity c independent of any object, i.e. its speed is c relative to any object irrespective of its velocity). But now comes the paradox: since the back of the train is moving towards the light beam and the front moves away from it, the two beams should reach the front and the end of the train not at the same time.

But which is it then? They can not both reach back and front simultaneously and at different times!

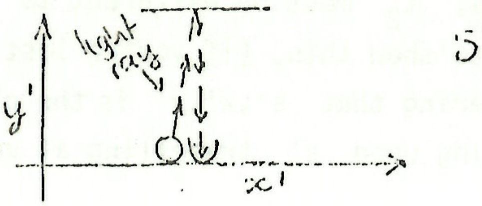
Or can they?

You see, that's the whole point of Einstein's theory: time, length and velocity are all relative measures in (and only in) respect to c. To you in the train, the beams do reach the two ends simultaneously. But to you on the hill, they do not - and this solely depends on the velocity of the train, V.

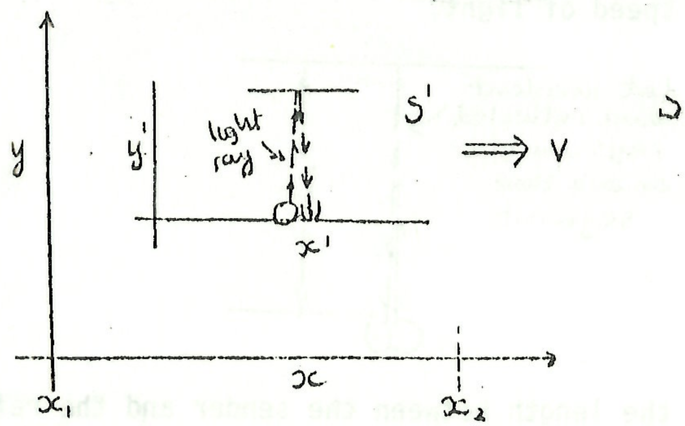
Thus, according to Einstein, the length of the train, or indeed, the time it takes for the beams to reach any given point, is not the same for you on the hill and you in the train. That is, we cannot compare two objects at different relative velocities, since their x,y,z, time co-ordinates are different. Hence we cannot say that two events are "simultaneous" because one hour for object A will be different for object B (although they will seem the same - here's another interesting aspect you might like to think upon), and this is shown by the comparison of A's watch to B's watch, which will show different times although they were originally indentically set.

OK, you say, so what? Lets take a simpler system and derive an expression.

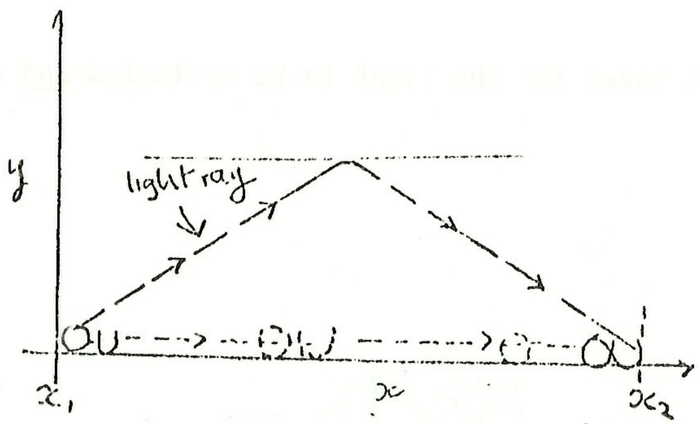
Call the closed system s', wherein it encompasses a device sending out a ray of light which is reflected from a mirror and is sent to a measuring time counter. Let horizontal and vertical displacements of this set-up be x', y' respectively.



Now imagine that system s' is given a horizontal velocity v relative to a stationary system s, with horizontal and vertical displacements x, y.



As s' moves from x_1 to x_2, s would see the movement of the system as such;



Now remember that velocity, $v = \frac{d}{t}$ and that in all cases, when $v = c$ the velocity of light stays constant for both observers in s' and s.

Thus for s', say $c = \frac{d_1}{t_1}$ and for s, say $c = \frac{d_2}{t_2}$. But for s, the ray of light has gone further than for s' (i.e. $d_2 > d_1$). Thus for c to remain the same value in both cases, t_2 must be different to t_1 ($t_2 > t_1$ actually), and we can derive a formula to show this, (if you're lost here, just go back over the last page or so, remembering that s',x',y' is the closed system and s,x,y is a stationary system looking upon s' travelling at velocity v).

Derivation

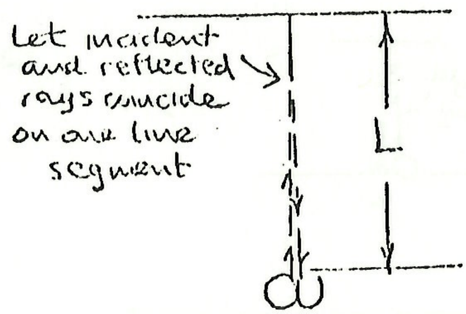
Let v = the velocity of system s' relative to s

t' = the time in the moving frame system s'

t = the time in the stationary system, s

c = the speed of light.

for s':



Lets say that the length between the sender and the reflection of the beam is such that it equals L.

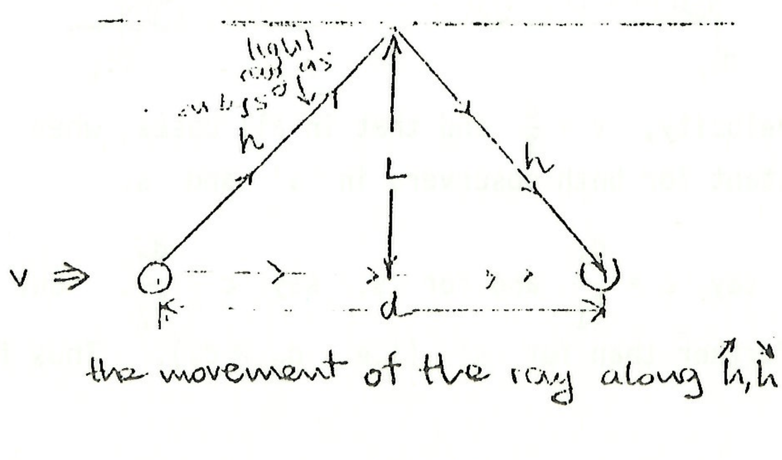
Now, $c = \frac{d}{t}$,

∴ the time it takes for the light to be reflected <u>and</u> returned is:

$$c = \frac{2L}{t'}$$

$$\therefore t' = \frac{2L}{C}$$

for s:



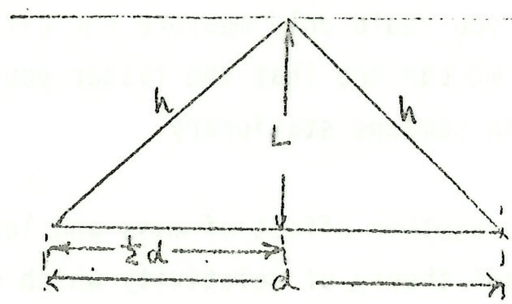
But for s, the distance travelled is greater; from the diagram:

$$c = \frac{2h}{t}$$

$$\therefore t = \frac{2h}{c} \quad (\text{where } h > L)$$

Also, from the diagram, $v = \frac{d}{t}$

$$d = vt$$



Now breaking our diagram up, we have:

$$h^2 = (\frac{d}{2})^2 + L^2$$

but we have derived these:

$$t = \frac{2h}{c} \Rightarrow h = \frac{ct}{2}$$

$$d = vt \Rightarrow \frac{d}{2} = \frac{vt}{2}$$

$$t' = \frac{2L}{c} \Rightarrow L = \frac{ct'}{2}$$

$\therefore$ substituting gives:

$$(\frac{ct}{2})^2 = (\frac{vt}{2})^2 + (\frac{ct'}{2})^2$$

i.e. $t = \dfrac{t'}{\sqrt{1 - \frac{v^2}{c^2}}}$ which is Einstein's time dilation formula.

Now lets go back to our train: imagine it is going at 36,000 m/s (yes, very fast) and the light beam takes one second relative to the man on the train to reach the end of the carrage (yes again, a very long train).

$$t = \frac{1}{\sqrt{1 - \frac{36{,}000^2}{(3 \times 10^8)^2}}}$$

$$\doteqdot 1.000000014\ldots$$

i.e., the 1 second for the man on the train would be approximately 1.000000014 sec for you on the hill, and you could only measure the difference by comparing your watches. Furthermore, we can see that the faster you go, the slower time would pass for you relative to someone stationary.

And this, in addition to another effect of mass and length contraction, makes the basis of Einstein's special theory of relativity which describes the effects on objects travelling at different linear constant speeds. And if this article gets published, I'll write a further continuum to the theory (it gets better, I promise).

Comments by Simon Prokhovnik, School of Mathematics, University of New South Wales:

I imagine that many readers of Ronnie Braverman's contribution on Special Relativity will feel that their fears in regard to this theory are indeed justified. This is not the fault of the article, nor of the author, because it is true that Special Relativity is by no means "the most complicated theory in the history of mankind", and, in fact, any regular reader of Parabola would have no trouble in understanding its mathematical formulation. The problem lies in the meaning of this formulation and of its consequences, for example, the reciprocity of observation (of length-contraction and time-dilation, etc.,) between two observers associated with different inertial reference frames, e.g. between an observer on the uniformly-moving train and an observer on the station platform.

There have been a number of reactions to this problem of interpretation, and the story of the resultant on-going debate is discussed in my book "The Logic of Special Relativity" (2nd ed., University of N.S.W. Press, 1978). However the difficulties and alleged paradoxes which underlie the debate are no reasons for avoiding one of the most beautiful and innovative mathematical theories ever produced.

Further, Special Relativity is a very important theory, not only because it entails the mass-energy equivalence result

$$E = mc^2,$$

with its implication that matter exists in two (interacting) forms - as 'mass-particles' and as 'energy'; but also because it determines the formulation of mathematical laws governing the physical world.

So I welcome Ronnie Braverman's contribution with a much enthusiasm as he himself displays, and hope that it will perhaps encourage a discussion in the pages of Parabola on the mathematics (e.g. its 'group' property) and the meaning of Special Relativity and the problems it raises. General Relativity - Einstein's theory of gravitation - is another matter again.

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OLYMPIAD NEWS

The second IBM training school for Olympiad students will be held at St. John's College, University of Sydney from May 12 to May 19 this year. This is the final stage of the preparation for the 1985 International Mathematical Olympiad to be held in Finland in July.

The six team members selected are:

Shane Booth: Wanganui Park High School, Victoria.
John Graham: St, Ignatius College, New South Wales.
Alasdair Grant: Melbourne Church of England Grammar, Victoria.
David Hogan: James Ruse High, New South Wales.
Andrew Hassell: Christ Church Grammar School, Western Australia.
Catherine Playoust: Loreto Convent, New South Wales.

Six other students will be attending the school in preparation for later Olympiads although selection for the Training School does not guarantee selection in future Olympiad teams.

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