Parabola Volume 21, Issue 2 (1985)

THE WORLD OF MATHEMATICS
RANDOM NUMBERS

BY

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There is a very old anecdote about probability that goes as follows:

"One day in Naples the reverend Galiani saw a man from the Basilicata who, shaking three dice in a cup, wagered to throw three sixes; and, in fact, he got three sixes right away. Such luck is possible, you say. Yet the man succeeded a second time, and the bet was repeated. He put back the dice in the cup, three, four, five times, and each time he produced three sixes. 'Sangue di Bacco,' exclaimed the reverend 'the dice are loaded!' And they were. Yet why did the reverend, use such profane language?"

The reverend observed that the sequence of fifteen throws turned up the numbers

6,6,6;6,6,6;6,6,6;6,6,6;6,6,6.

There are two reactions possible to such an outcome: to be surprised, or not to be surprised. Some people would argue that there is nothing to worry about, since any combination of fifteen numbers is as likely as any other. The chances of getting fifteen 6's in a row is only $(1/6)^{15}$, but this is equal to the chance of getting, say

3,5,1;4,2,4;3,6,6;5,3,1;6,4,1.

This sequence would not surprise anyone. In the same way, it is unlikely that any given person will win Lotto, but someone has to win it, and whoever it is, we are not surprised. On the other hand, there does seem something to be said for the reverend Galiani's feeling that 3,5,1;4,2,4, etc. would have been all right, since it is "random", but that 6,6,6;6,6,6 etc. is just not random enough.

What then is this quality of "randomness" that 3,5,1;4,2,4.... has but 6,6,6;6,6,6... lacks? What are tables of "random numbers?" Can computers generate random numbers? Are the digits of π random? To answer these questions, or even to make sense of them, it is necessary to begin by distinguishing two senses of "randomness". In the first sense, a sequence of digits is called random (probabilistic) if it is produced by some probabilistic or chance process. Thus if fair dice are thrown and the numbers that appear are written down, the

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sequence that results is random (probabilistic) whether it "looks random" or consists entirely of 6's. In the second sense, a sequence of digits is called random (patternless) if it has no pattern or order. Thus a sequence of 6's only is not random (patternless), nor is a sequence that repeats, such as 1,3,4,1,3, 4,1,3,4,1,3,4,1,3,4. The connection between the two concepts is that a random (probabilistic) sequence is almost always random (patternless). This explains why the reverend Galiani was surprised by fifteen 6's when he would not have been surprised by a patternless sequence: highly ordered sequences form a very small proportion of all sequences of a given length. So we are surprised if an ordered one appears.

Computers cannot generate tables of random numbers, in the probabilistic sense, since they always use algorithms which are totally deterministic. They do generate random (patternless) sequences, and in fact computers are <u>better</u> at doing this than dice - dice occasionally produce ordered sequences, but random number generating algorithms are guaranteed to produce sequences which are random (patternless).

Are the digits of π random? The answer is not known for certain. The decimal expansion of π begins:

3.14159265358979323846264338327950288419716939937....

These digits certainly look random. The first million digits of π have been written down, and these too appear to be random. This gives good reason to believe that the digits of π will go on being random indefinitely, but this hypothesis has not been proved. In fact there does not even exist a totally satisfactory definition of "random (patternless)", and this is still an area of active research by mathematicians and computer scientists.

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