PROBLEM SECTION

Q.536. In the play off match for the chess club championship between the three players who had finished level after the preliminary tournament, each pair played the same number of games. Reporting on the match, the club news sheet stated "A won most games, B lost fewest times, but C won the championship with the most points." Could the report be correct?

Q.537. Is it possible to dissect a triangle into concave pentagons? (i.e. pentagons with re-entrant angles).

Q.538. Tiddlyball is a three player game. In each round, the winner scores a point, the runner up is awarded b points, and the loser gets c points, where $a > b > c$ are positive integers. One day Xavier, Yvonne & Zachary played some tiddlyball and the final score was

Xavier - 20 Yvonne - 10 Zachary - 9.

Yvonne won the second round. Who won the first round, and how many points did Zachary get in the last round ?

Q.539. For a positive integer n, let n^* be the number which results by writing n to the base 2 and then reading the result as though it were written in base 3. For instance, if $n = 6$, then $6 = 110_2$ and $110_3 = 12$ so that $6^* = 12$. Find all numbers n such that $n^* = 9n$ and prove there are no others.

Q.540. The numbers $1,2,\ldots,64$ are written onto the squares of an 8×8 chessboard. Prove that there is a pair of adjacent squares which contain numbers that differ by ≤ 16. (Squares are adjacent if they share a corner, or a side).

Q.541. Does there exist an infinite sequence of perfect squares such that the sum of the first n elements is also a perfect square for every n ?

Q.542. A certain pentagon has the property that each of the 5 lines joining a vertex to the mid-point of the opposite side bisects the pentagon into two parts of equal area. Prove that these five lines are concurrent.

Q.543. The triangle ABC is right angled at A, and the lengths of AB, AC are 3,4 respectively. It is possible to draw two squares inside the triangle all of whose vertices lie on the sides of the triangle. Find the area of the overlap region inside both squares.

Q.544. Let S be a set of m real numbers ($m > 2$) with the property that the absolute value of each number in S is not less than the absolute value of the sum of all the others. Prove that the sum of all the numbers in S is zero.

Q.545. Let $\square$ be an operation which combines two integers to give a new integer. Suppose that

$$x \square (y+z) = (y \square x) + (z \square x)$$

for all x, y, z. Prove that

$$u \square v = v \square u \quad \text{for all } u, v.$$

Q.546. I put 62 stones on an ordinary 8×8 chessboard putting one in each square and leaving two opposite corners empty. I now remove stones from the board by jumping two at a time and removing the two jumped over. Thus if I have

abcd

I can jump the stone in square a onto d, removing b and c and leaving

Jumps can be horizontal or vertical but not diagonal.

Prove that if I keep jumping until there are just two stones left, they must be on squares of the same colour.