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# MATHEMATICS OF SIMPLE CAR-PARKING*

by

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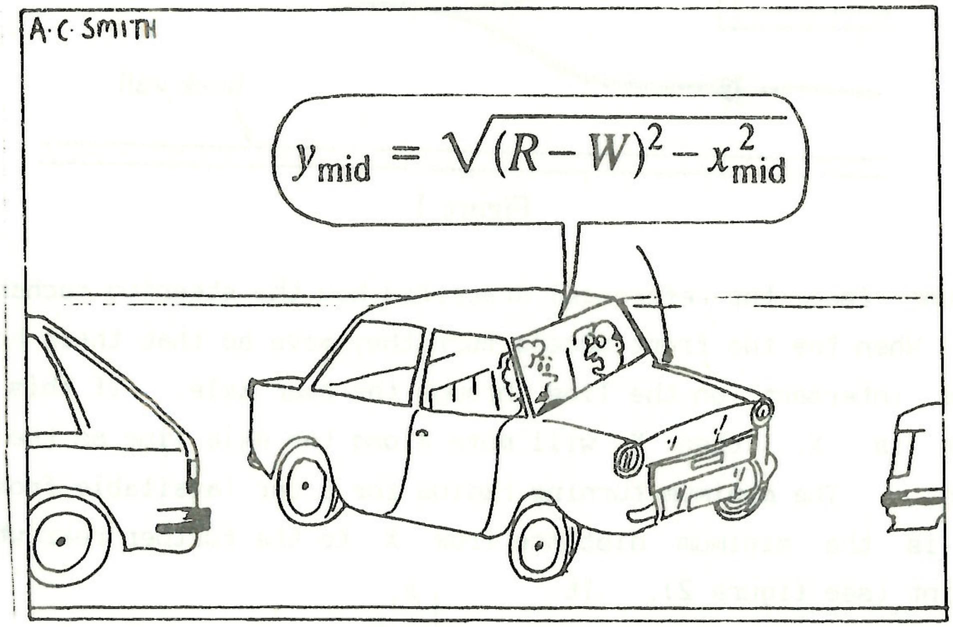


Parking a car constitutes a bad dream for some drivers. It is really a matter of common sense and simple geometry, but some (especially this author) tend to panic at the wheel, and turn relatively simple manoeuvres into a nightmare.

We park our car on a concrete area behind the house, and access is via a narrow entry. Usually I drive up the entry and reverse on to the concrete area, and reverse the procedure to come out. However, this involves reversing out on to a fairly busy road. The other week I tried the operation the other way round, reversing up the entry and driving forward into the parking space. The result was that I had a nasty encounter with a wooden post. Subsequently, to get out was a problem too! Calculating the mathematics of 'how' is easier, and safer, than experimentation, and these are some of the results.

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Figure 1 shows the parking space and obstacles, together with an indication of the usual mode of entry.

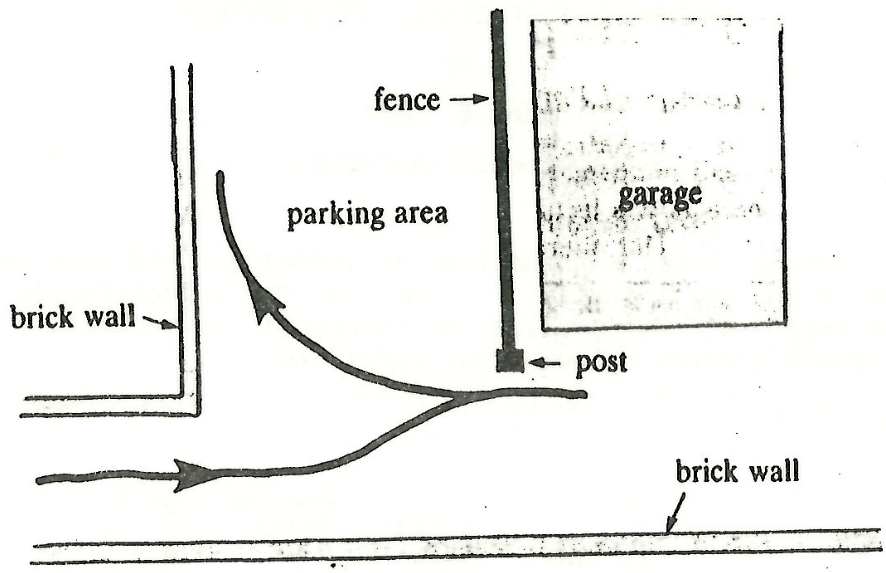


Figure 1

Sid Dunn (see the reference) describes how the steering mechanism of a car works. When the two front wheels turn they move so that their horizontal axes always intersect on the line through the rear axle. If this point of intersection is X, then X will move along the axle line as the steering wheel turns. The minimum turning radius for a car (available from the car hand-book) is the minimum distance from X to the further rear wheel road contact point (see figure 2). It determines the maximum steering capacity of the car Once the steering wheel is on full lock the car will move in such a way that the further rear wheel travels in a circle centre X and radius R. X is constant until the steering wheel is moved again. Let the minimum turning radius be R, the length of the car be L and the width of the car be W.

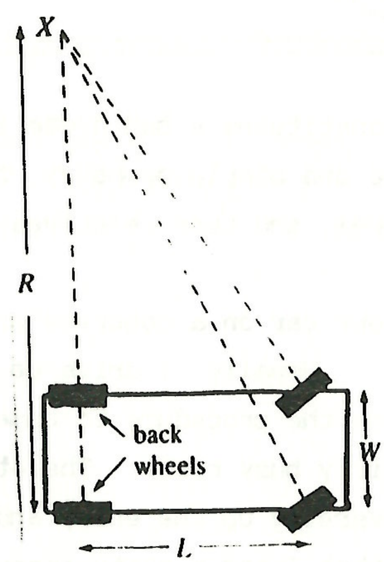


Figure 2

Our problem is to turn the car through $90^o$. We shall consider reversing into the parking space. If the steering wheel is on full lock then the further rear wheel travels on the circumference of a circle radius R, the nearer rear wheel travels on the circumference of a circle R - W and the further front wheel travels on the circumference of a circle radius $\sqrt{R^2 + L^2}$ (see figure 3).

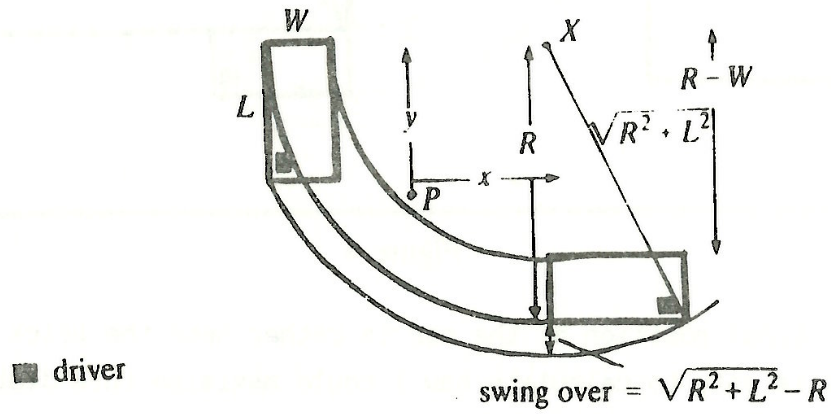


Figure 3

The care therefore covers the shaded area between two circles, and for a comfortable ride this must be obstacle-free. As the car turns, the front wheel swings over by an amount $\sqrt{R^2 + L^2} - R$, as shown, so the car must initially be at least this distance away from the brick wall. Now the wooden post P is fixed in space, but X is determined by the initial position of the car. Once I turn the wheel on full lock then X remains fixed. I therefore need to decide on the position of X relative to P in order to manoeuvre the car successfully. This position of X then defines the starting position of the car. The post P must lie inside the inner circle. If its coordinates relative to X are (x,y), then $x^2 + y^2 < (R - W)^2$. If $w_1$ and $w_2$ are widths of the entry and parking space respectively, them $w_1$ must be at least $W + \sqrt{R^2 + L^2} - R = w_{min}$. If $w_1 = w_{min}$ then $w_2 > R$.

My usual parking solution is to position the back of the car in line with the fence, and turn the wheel on full lock (see figure 4, $X = X_0$).

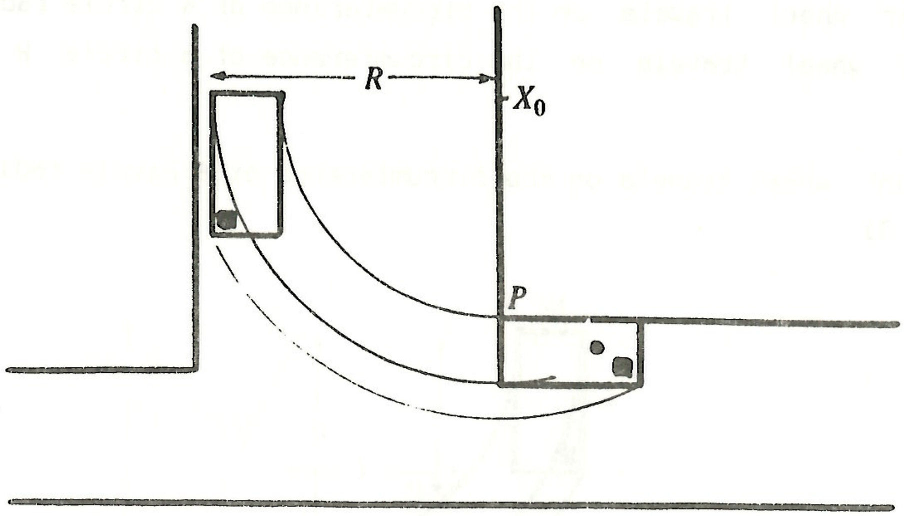


Figure 4

However, the final position of the car is rather near the brick wall. Using our results it is easy to calculate how I could maximise this distance from the wall (to make it easier to get out of the car) or position the car in the centre of the concrete area.

In order to finish away from the wall I need to choose $X_1$ in order to maximise x, i.e. to minimise y (see figure 5).

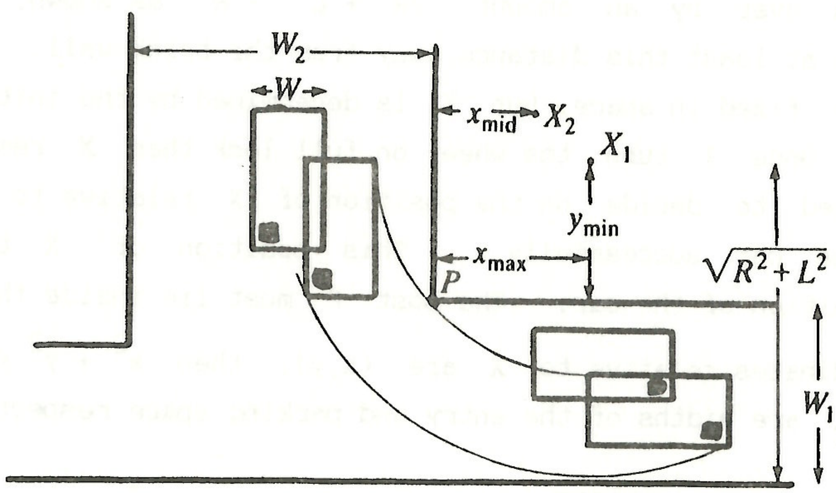


Figure 5

Now

$$y_{min} = \sqrt{R^2 + L^2} - w_1 \quad \text{and so} \quad x_{max} = \sqrt{(R - W)^2 - y_{min}^2}.$$

I should need to drive past the post until the back wheels were $x_{max}$ away from it, I should need to drive past the post until the back wheels were $x_{max}$

away from it, and also position the car as near the wall as possible, i.e. distance $\sqrt{R^2 + L^2} - R$ away from it.

In order to place the car in the centre of the parking space, I need

$$x_{mid} = R - \frac{1}{2}(W + w_2), \quad \text{so} \quad y_{mid} = \sqrt{(R - W)^2 - x_{mid}^2},$$

giving $X_2$, from which it is possible to calculate the starting position of the car, if feasible.

When driving forwards the analysis is similar, but in reverse (see figure 6). It should be clear why it is easier to reverse in, and also what mistake I made when I hit the post.

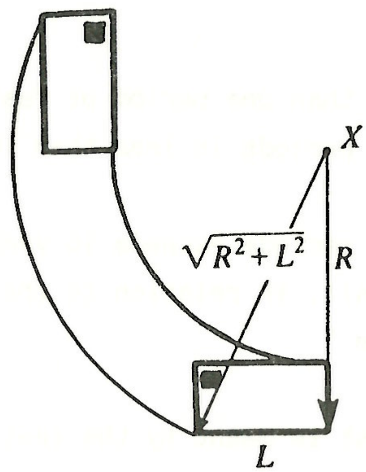


Figure 6

For those interested, Sid Dunn's chapter contains mathematics of more complicated manoeuvres, and further exercises for the enthusiast.

The above workings need slight adjustment to cater for the body of the car. Incidently, since this analysis I have had little problem in parking the car!

REFERENCE


S.C. Dunn, Parking a car, in Case Studies in Mathematical Modelling, ed. D.T.G. James and J.J. McDonald, pp. 110-123 (Stanley Thornes, Cheltenham, 1981).


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