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PROBLEM SECTION

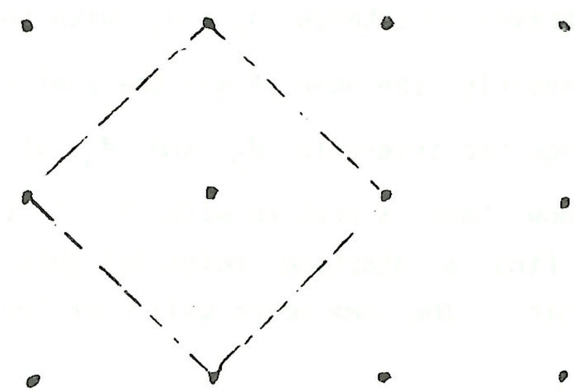
You are invited to submit solutions to one or more of the following problems. Solutions will appear in Volume 23 No. 1

Q. 672. Find the least natural number whose last digit is 6 such that it increases by the factor 4 when this last digit is carried to the beginning of the number.

Q. 673. What are the last 2 digits of $14^{14^{14}}$?

Q. 674. Let A and B be any 2 different 7 digit numbers each of which is composed of all the digits from 1 to 7. Prove that neither is a multiple of the other.

Q. 675. In the accompanying 3 × 4 array of equally spaced dots there are altogether 10 sets of 4 dots which lie at the vertices of a square. (One such set is shown by the dotted square in the figure. There are 6 small squares, 2 larger ones and 2 at an angle.) Find for an m × n array a formula for the total number of squares all of whose vertices are in the array.



Q. 676. $x_1, x_2, \ldots, x_{19}$ are positive integers with the property that if any one of them is deleted it is always possible to partition the remaining eighteen into two sets of nine numbers with equal sums. Prove that all nineteen numbers must be equal.

Q. 677. Students from forms V and VI took part in the school chess tournament. Each entrant played one game with every other participant, scoring one point for a win, half a point for a draw, 0 for a loss. The number of participants from form VI was 10 times the number from form V, and the points gained by all from VI students together was 4.5 times the number of points earned by the form V contingent. How many students from each form took part?

Q. 678. The numbers 1, 2, 3, ..., n^2 are arranged in a square array as in the figure. One of the numbers s_1 is placed in a set S and the row and column containing s_1 are deleted from the array. Any remaining number, s_2, is now placed in S and the column, and row containing s_2 is deleted. This process is continued until only one number, s_n remains. Find the sum of the numbers $s_1 + s_2 + \dots + s_n$ and prove your result.

1	2	....	n
$n+1$	$n+2$		$2n$
$2n+1$	$2n+2$		$3n$
....	....	....	....
....	....	....	....
....	....	....	....
$(n-1)n+1$	....	....	n^2

Q. 679. We are given a rectangular array of numbers arranged in m rows and n columns. We are permitted to change the sign of every number in any row, or to change the sign of every number in any column. Prove that if we perform these admissible operations repeatedly we can eventually obtain an array in which the sums of the numbers along every row and down every column are all non-negative.

Q. 680. Show that if p is any odd prime number except 5, then there are values of k such that the number 1 1 1 ... 1 composed of k ones is a multiple of p.

Q. 681. Let $x_1, x_2, x_3, \dots$ be any infinite list of digits not containing the digit 9. Consider the list of numbers $y_1, y_2, y_3, \dots, y_n, \dots$ where y_n has decimal representation $x_1 x_2 \dots x_n$. (i.e. $y_n = x_n + 10x_{n-1} + 10^2 x_{n-2} + \dots + 10^{n-1} x_1$). Prove that this list contains infinitely many composite numbers.

Q. 682. $P(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ is a polynomial with integer coefficients such that $P(2) = 7$ and $P(5) = 1$. Prove that the equation $P(x) = 0$ has no integer roots

Q. 683. i) Let A be any number however large. Prove that for n sufficiently large $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} > A$.

ii) If all terms whose denominators contain the digit 7 are deleted from the sum $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$ prove that the total of the remaining terms is less than 80.

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