PROBLEM SECTION

Q. 684. $\hat{BAC}$ is an obtuse angle. A circle through A cuts AB at P and AC at Q. The bisectors of angles $\hat{QPB}$ and $\hat{PQC}$ cut the circle at X and Y respectively. Prove that XY is perpendicular to the bisector of $\hat{BAC}$.

Q. 685. Solve the equation

$$9^{x+1} - 5^{2x+1} = 3^{2x-1} + 5^{2x-1}.$$

Q. 686. Both c_1 and c_2 are expressible in the form $c = x^2 + 3y^2$ where x and y are whole numbers. Prove that their product c_1c_2 is also expressible in that form.

[e.g. $c_1 = 13 = 1^2 + 3.2^2$; $c_2 = 12 = 3^2 + 3.1^2$ and $156 = 12.13 = 9^2 + 3 \times 5^2$.]

Q. 687. Find all integers x,y,z such that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{x+y+z}.$$

Q. 688. A regular polygon with n vertices is inscribed in a circle of radius 1. Let L be the set of all mutually distinct lengths of all line segments joining the vertices of the polygon. What is the sum of the squares of the elements of L?

Q. 689. N is a number less than 500 with three distinct digits, none of them 0. Five different numbers car be obtained by changing the order of the digits of N. The arithmetic mean of these five numbers is equal to N. Find N

Q. 690. i) Is there a natural number n such that $n^2 + n + 1$ is exactly divisible by 1985, or by 1986?

ii) Is there a natural number n such that $2(n^2 + n + 1)$ is divisible by 1986?

Q. 691. Add 3 digits to the end of 523 so that the resulting 6 digit number is divisible by 7,8, and 9.

Q. 692. Seven colours of paint are available. In how many ways can the 6 faces of a rectangular block be painted if faces meeting at an edge must be given different colours.

Q. 693. N is a power of 2, and $(a_1, a_2, \ldots, a_N)$ is a sequence of N numbers each equal to either +1 or −1. Another sequence (b_i) of the same length is obtained by the following operation:-

$$b_i = a_i a_{i+1} \quad i = i, \ldots, N - 1 \quad ; \quad b_N = a_N a_1 .$$

This operation is performed repeatedly. Prove that after enough repetitions of the operation all terms of the resulting sequence are equal to +1.

Q. 694. A "Latin square" of order n is a square array of n lines and columns each of which consists of an arrangement of the same n symbols. The figure is an example of a Latin square of order 3 (using the symbols 1, 2, 3). It is a "symmetric" Latin square i.e. symmetric about the "main" diagonal from the 1st element of the top row to the last element in the bottom row. This means that for each i,j the element in the ith row and jth column is the same as that in the jth row and ith column.

1	2	3
2	3	1
3	1	2

Prove that on the main diagonal of every symmetric Latin square of odd order each of the n symbols occurs once.

Q. 695. Suppose $p(x)$ denotes a polynomial $a_0 + a_1x + \ldots + a_nx^n$ which is such that the equation $p(x) = x$ has no real roots. Show that the equation $p(p(x)) = x$ also has no real roots.

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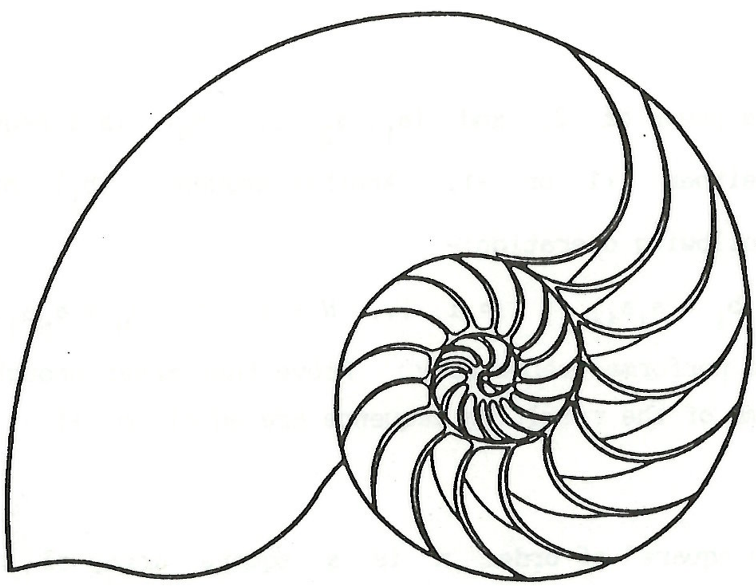

The Nautilus Spiral