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UNDERPROMOTIONS IN CHESS

by

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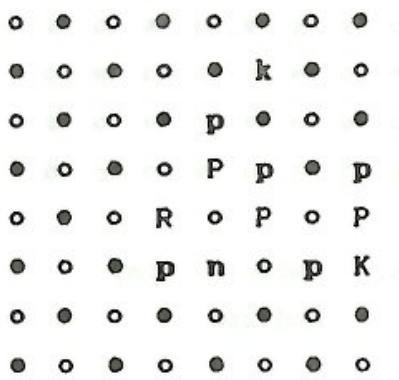
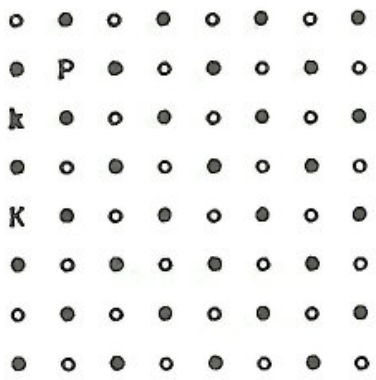
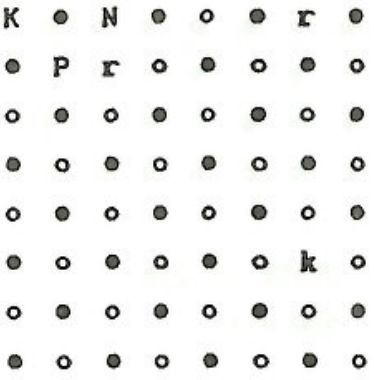
It is not easy to construct chess positions in which it is rational to advance a pawn to the eighth rank and promote it to a rook, knight or (especially) a bishop. The July 1985 issue of *Chess in Australia* contained that rarest of phenomena in competitive chess: a pawn that has to be promoted to a bishop. This is diagram 1 below.



1. Black to play and win.



2. White to play and win.



3. White to play and draw.

(All diagrams are given from White's viewpoint. The bottom left square is denoted by a1, the bottom right by h1, etc. The White pieces are indicated by capital letters. The Black pieces are indicated by lower case letters.)

In diagram 1, Black (C. Depasquale) won by 1. ... g2!; 2. R:d3(!) g1(B)!!. The point is that if the pawn is made into a queen White can force either perpetual check or stalemate whilst neither rook nor knight can win from this position. Thus the pawn was promoted to a bishop to deprive the inferior side of stalemate possibilities. A well-known position (diagram 2 above), in which Black has presumably just played K(a7):a6, shows a variation of this theme. White can win only by playing 1. b8(R) with checkmate to follow in a few moves.

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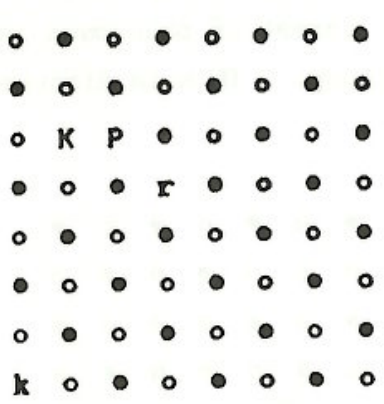
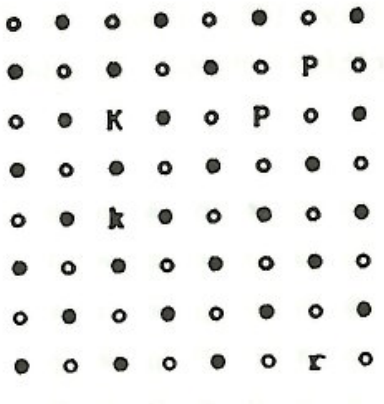
Diagram 3 is the climax of a composed position. Here White is the inferior side and cannot get away with 1. b8(Q)? Rg:c8. However after 1. b8(B)! Rg:c8 is stalemate and any other try by Black gives a simple draw to White.

There is an immense literature on chess curiousities, including perhaps 100,000 published "endgame studies". These are contrived positions in which surprising moves are used to achieve a win or a draw. The editor has allowed the once-off publication of this pseudo-mathematical material as an introduction to the magic that generations of ingenuity has found on the chess-board.



4. White to play and win.

5. White to play and win.

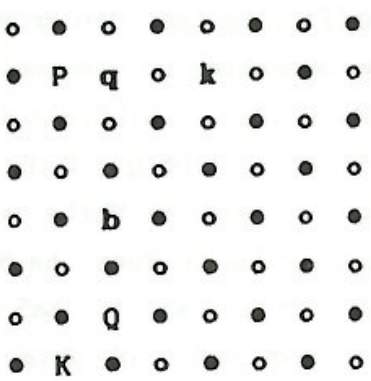


6. White to play and win.

The origin of diagram 4 is in dispute. One reference attributes it to Saavedra (1895) whilst another attributes it to an amateur game played in the year with the correct play being discovered by E. Lasker, the algebraist and philosopher who was World Chess Champion from 1895 to 1921. The best play is: 1. c7 Rd6+: 2. Kb5 Rd5+; 3. Kb4 Rd4+; 4. Kb3 Rd3+; 5. Kc2 Rd4(!): 6. c8(R)! (avoiding 6. c8(Q) Rc4+; 7. Q:c4 stalemate) Ra4; 7.Kb3 and White wins.

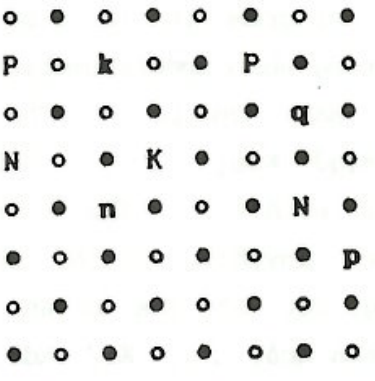
Diagram 5 is well known. White can win with a forking check from a new knight. 1. Rc8+ R:c8; 2. Qa7+ K:a7; 3. b:c8(N)+.

Diagram 6 is taken from page 278 of *Basic Chess Endings* by R. Fine, and nearly arose in professional play. The key line is: 1. Kd7 Kd5; 2. Ke8 Ke6; 3. f7 Ra1(!); 4. f8(N)+! This check saves a tempo and so prevents 4. ... Ra8 checkmate. The next White move is 5. g8(Q).



7. White to play and win.

8. White to play and mate in 8.

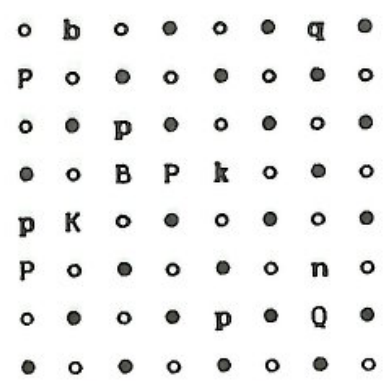


9. White to play and win.

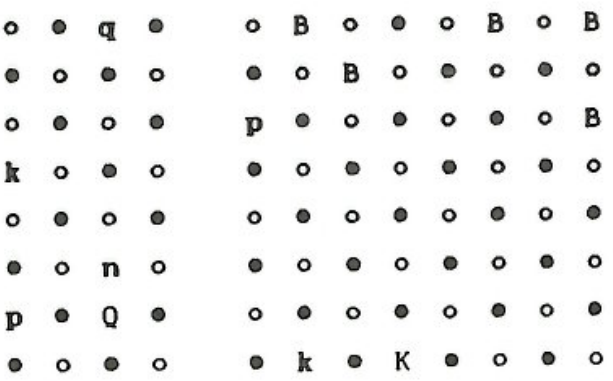
Diagram 7 is a more refined example of the tempo-saving knight promotion. It was published by the Russian composer A. Troitsky in 1901. White wins the Black queen as follows: 1. Qh7+ Bf7; 2. Qh4+ Ke6; 3. Qh3+ Kd5; 4. Qb3+ Kc6; 5. b8(N)+ Kd6; 6. Qg3+.

Diagram 8 was published by A. Cheron in a Swiss newspaper in 1964. It features eight successive promotions of pawns to knights. 1. b8(N)+ R:b8; 2. a:b8(N)+ Kd6; 3. c8(N)+ Ke6; 4. d8(N)+ B:d8; 5. e:d8(N)+ Kf6; 6. g8(N)+ R:g8; 7. h:g8(N)+ Kg6; 8. f8(N) checkmate.

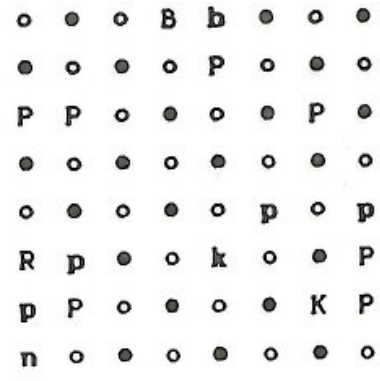
Diagram 9 was published by Troitsky in 1912. The Dover edition of his endgame studies omits the solution "since any attempt to solve the ending would merely result in failure and frustration for the would-be solver". However the first few moves are relatively obvious: 1. a8(N)+ Kd7; 2. f8(N)+ Ke8; 3. N:g6 Ne3+(!); 4. N:e3 h2. The position now changes into the unusual endgame of king plus four knights versus king and queen. Presumably Troitsky was able to demonstrate a win for White.



10. White to play and win.



11. White to play and win.



12. White to play and win.

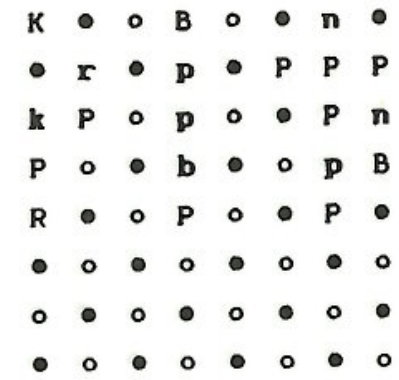
Diagram 10 is also due to Troitsky and shows the difficulty that can arise in winning when underpromotion results in a surfeit of bishops operating on diagonals of the same colour. The principal line begins: 1. Q:g3+ Q:g3; 2. a:b8(B)+ K:d5; 3. B:g3 Ke6; ... . Note that Black would have drawn after 2. a:b8(Q)+ K:d5; 3. Q:g3 e1(B)+. The point now is that if the Black king can reach a8 White cannot force anything better than stalemate. Both bishops are needed to keep the Black king out of the haven square. The principal line continues: 4. Ka5 Kd7; 5. Kb6 Kc8; 6. Ka7 Kd8; 7. Kb8 Kd7; 8. Kb7 Kd8; 9. K:c6 Kc8; 10. Bcd6 Kd8; 11. Bh4+ Kc8; 12. Bh2 e1(Q); 13. B:e1 Kd8; 14. Kb7 Ke7; 15. Bh4+ Kd7; 16. Bg5 Ke6; 17. Kc6 Kf5; 18. Kb5 and White wins.

Diagram 11 shows what can be done with even more bishops created by promotion. It is said to be due to Troitsky. All five bishops are needed in the following main line: 1. Bce5 a5; 2. Ba1 a4; 3. Bbe5 Ka2; 4. Kc2 a3; 5. Kc3 K:a1; 6. Kb3+ Kb1; 7. Ba1 a2; 8. Kc3 K:a1; 9. Kc2 checkmate.

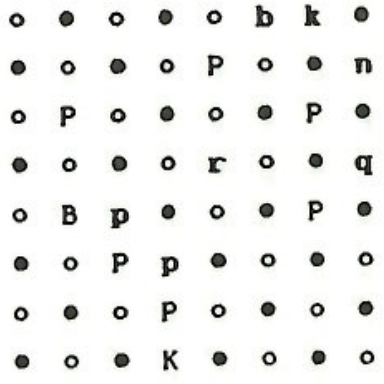
Diagram 12 was published by Troitsky in 1923 and shows White winning by three different underpromotions. The knight stops checkmate whilst the rook and bishop promotions prevent potential stalemates. 1. b7 f3+; 2. Kh1 f2; 3. Bb6+ Ke2; 4. B:f2 K:f2; 5. b8(N) Nc2; 6. Ra5 Na3(!); 7. R:a3 B:g6; 8. e8(R) B:e8; 9. a7 Bg6; 10. a8(B) and White wins.

The most difficult task in finding positions where underpromotion is needed would appear to be in finding examples which features multiple promotion to bishops. Indeed Cheron's position 8 above (with 8 promotions to knights) is relatively crude. I believe that he found a companion position with 8 promotions to rooks. More information on promotion to bishops is given by H.M. Lommer's *1357 End-game Studies* (1975), which is a sequel to his *1234 Modern Endgame Studies* (1937).

13. White to play and win.



14. White to play and win.



15. White to play and win.

Position 13 is due to V.A. Korolkov (1941) and needs three under-promotions of pawns to bishops to avoid stalemate. The main line is: 1. a8(B)! Nc4; 2. d:c4 d3; 3. e5 B:a8; 4. c8(B)! Bf3; 5. Bb7 Nc6; 6. B:c6 B:c6; 7. g8(B)! Bf3; 8. Bd5 and White wins. A few years later Lommer came up with position 14, which needs four underpromotions to bishops! The main line here is 1. f:g8(B) N:g8; 2. h:g8(B) B:g8; 3. B:g5 Bd5; 4. g8(B) B:g8; 5. g7 Bd5; 6. g8(B) B:g8; 7. Bg6 and White wins. The point is that the four pawns had to get out of the way (by being bishoped) to let the existing bishop on h5 enter the game. The line White has to avoid is: 1. f:g8(Q)? Nf7!; 2. Q:f7 R:b6+; 3. Q:d5 Rb8+; 4. K:b8 stalemate. Position 15 is also due to Lommer. You may wish to calculate the optimal promotions of the e7 pawn after Black has responded to 1. g:h7+.

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