# PROBLEM SECTION

**Q.708.** A four digit number a b c d has the property that $a + b = c \times d$ and also $a \times b = c + d$. Find all possibilities.

**Q. 709.** Prove that the numbers 49, 4489, 444889, .... (each obtained by inserting 48 into the middle of the preceding number) are all perfect squares.

**Q.710.** A number is "palindromic" if it reads the same with the digits in reverse order (e.g. 42724). Find all palindromic numbers with 6 digits which are obtainable by adding two palindromic 5 - digit numbers.

**Q.711.** One obvious solution of the equation $(1.001)^x = 1 + x^{1000}$ is $x = 0$. There is another (large) value of $x$ which satisfies the equation. Can you find an approximate value of the second solution?

**Q. 712.** Given positive integers $a$ ($\neq 1$), $m$, and $n$, where $m$ and $n$ have no common factor except 1, find the highest common factor of $a^m - 1$ and $a^n - 1$ and prove your result.

**Q. 713.** You are given 100 points in a plane, no three of which are collinear. Any 50 are chosen and labelled A, the others being labelled B. Show that it is possible to draw 50 straight line segments each linking a point labelled A to a point labelled B in such a way that

(i) each of the 100 points is an end point of one line segment.

and (ii) no two line segments intersect.

**Q. 714.** I have four infinite sets of whole numbers $X_1$, $X_2$, $X_3$ and $X_4$. Every non-negative integer $m$ can be expressed uniquely in the form

$$m = x_1 + x_2 + x_3 + x_4$$

where $x_1 \in X_1$, $x_2 \in X_2$, $x_3 \in X_3$, and $x_4 \in X_4$.

(i) Show that 0 is in every set, but that no other number can be in more than one set.

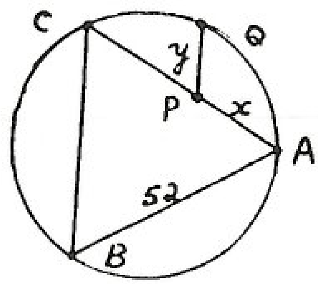
(ii) For any $N > 0$ let $n(N)$ be the number of elements of $X_1$ $X_2$ $X_3$ $X_4$ which are less than $N$.

Show that $n(N) \geq 4\,N^{1/4} - 3$ for any $N$.

(iii) Find four sets having the stated properties, and if possible a number $N$ for which

$$n(N) = 4\,N^{1/4} - 3.$$

Q. 715.




In the figure, A B C is an equilateral triangle, with sides 52 cms long, inscribed in a circle. PQ is parallel to BC, and the lengths AP, PQ are $x$ cms, $y$ cms respectively, where $x$ and $y$ are both whole numbers. Find all possible values of $x$ and $y$.

Q 716. (i) Suppose that $x$ is a real number with the property that there is an infinite sequence of rational numbers.

$$\frac{p_1}{q_1}, \frac{p_2}{q_2}, \ldots, \frac{p_n}{q_n}, \ldots$$

($p_n$, $q_n$ are integers), all different from $x$, but such that

$$\left| x - \frac{p_n}{q_n} \right| < \frac{1}{q_n^{\,2}} \quad \text{for } n = 1, 2, 3, \ldots.$$

Prove that $x$ cannot be a rational number.

(ii) Let $x = \frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^7} + \ldots + \frac{1}{2^{2^n-1}} + \ldots.$

Show that $x$ is an irrational number.

Q 717. (i) The function $f(x) = e^x = \exp(x)$ has the property that $f'(x) = f(x)$ for all $x$.

Prove that for all positive numbers $x$

$$e^x > 1 + x.$$

(ii) Let $x_1, x_2, \ldots x_n$ be any positive numbers,

$S_n = x_1 + x_2 \ldots + x_n$, $Q_n = (1+x_1)\times(1+x_2)\times \ldots \times(1+x_n)$.

Show that $\exp(S_n) > Q_n > S_n$

(ii) Let $a_k = \dfrac{k^2 + k + 1}{k^2 + k}$, $p_n = a_1 \times a_2 \times \cdots \times a_n$.

Show that $p_n < e$ for any positive whole number n.

**Q. 718.** For any positive integer n

$$x_n = (1 + \frac{1}{2}) \times (1 + \frac{1}{3\times2+3}) \times \ldots$$

$$\ldots \times (1 + \frac{1}{n\times(n-1)\times\ldots\times2 + n\times(n-1)\times\ldots\times3 + \ldots + n})$$

Show that $x_n < 2$ for all n.

**Q. 719.** For any integer n each of f(n), g(n), and h(n) denotes a corresponding integer. No two of f, g, and h are identical functions. (e.g. There is at least one integer m such that $f(m) \neq g(m)$).

Each of f, g, and h satisfies the functional equation:-

$$F(mn) = F(m)\,F(n) + F(m+n) - 1 \quad \text{for all integers } m, n.$$

(e.g., $h(mn) = h(m)\,h(n) + h(m+n) - 1$ for all integers m, n).

If $f(1987) = f(1)$ and $g(1987) = g(0)$, find $h(1987)$.

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ANSWER TO THE PROBLEM ON PAGE 11

Pythagoras daughter is 9 years old.