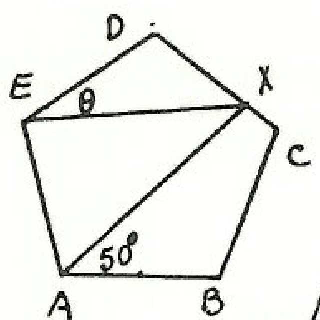
PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of Parabola; your solution(s) may be used if they are received in time.
Q.759 is the corrected version of Q.745 promised in the solutions section. Mr. C. Buecher (Airds HS., Campbelltown) proposed problems Q.755 and Q.756. (I have taken the liberty of changing the notation in the latter problem.)

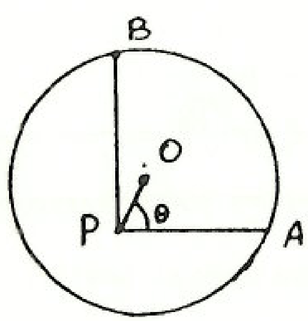
Q.753 When multiplying two whole numbers a student by mistake reduced the tens digit in the answer by 7. He checked his answer by dividing it by the smaller factor, obtaining the quotient 48 and the remainder 17. Find the two factors.

Q.754 A bath was being filled with water from two taps. The cold tap was first turned on for a period equal to one fifth of the time required to fill the bath by the hot tap alone. Then it was turned off and the hot tap was run for one-fifth of the time required to fill the bath by the cold tap alone. By now the bath was 5/12 full, and it required another 5 minutes to complete filling the bath with both taps turned on.

Find how long each tap separately takes to fill the bath.



Q.755 $ABCDE$ is a regular pentagon, and X is the point on CD such that $X\hat{A}B = 50°$. Calculate $D\hat{E}X$.

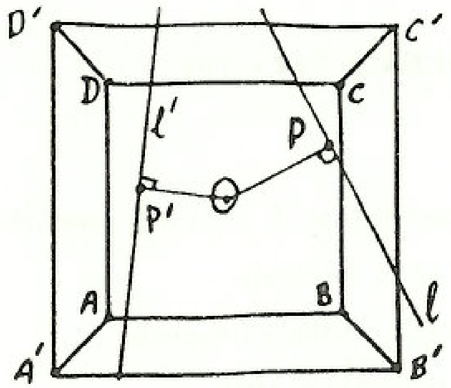


Q.756 The figure shows a circle, centre 0, radius 1. P is an interior point and A, B are points on the circumference such that $A\hat{P}B = 90°$ and such that exactly half the area of the disc lies within the angle.

(i.e. the area shaded $= \frac{1}{2}\pi$). Let $OP = r$ and $O\hat{P}A = \theta$.

(i) Find the range of possible values of r.

(ii) For r in that range, find an expression relating r and θ.



Q.757 The figure exhibits two squares $ABCD$ and $A'B'C'D'$ with a common centre 0, and with AB parallel to $A'B'$. A line ℓ cuts across the squares but does not intersect any of the intervals AA', BB', CC' or DD'. P is the foot of the perpendicular from 0 to ℓ.

(Two possible positions of ℓ and P are shown in the figure).

(i) Describe the region which is the locus of P.

(ii) Calculate its area in terms of the side lengths of the two squares.

Q.758 Let $f(x) = x^n$ where n is a fixed positive integer and $x = 1, 2 \cdots$ Define a by its decimal expansion $a = 0.f(1)f(2)f(3)\cdots$ For example if $n = 5$ the first few digits in a are $a = 0.13224310243125\cdots$ using the digits in order of $1^5, 2^5, 3^5, 4^5, 5^5, etc.\cdots$ Is it possible to choose n so that a is a rational number?

Q.759 A function $f(n)$ is defined for positive integers n in such a way that $f(1) = 1, f(2) = 2$, and if $3^{m-1} \leq q < 3^m$ and $r = 0$,1 or 2 then $f(3q + r) = r3^m + f(q)$.

For how many values of n between 1788 and 1988 is $f(n)$ equal to n?

Q.760 The real numbers x and a are related by $\sqrt{x^2 - a^2} + 2\sqrt{x^2 - 1} = x$. Find the range of possible values of a and solve for x when a is in that range.

Q.761 (i) Let p denote the perimeter of a triangle, and r the radius of the inscribed circle. Prove that $r \leq \frac{p}{6\sqrt{3}}$.

(ii) $P_1, P_2 \cdots P_n$ are the verticies of an n-gon inscribed in a circle. The centre O of the circle lies inside the n-gon. Let A denote the sum of the areas of the circles inscribed in the n triangles $\Delta P_1OP_2, \Delta P_2OP_3, \cdots \Delta P_nOP_1$, and let B denote the area of the n-gon. Show that $\frac{A}{B} \leq \frac{\pi}{3\sqrt{3}}$. If $0 < k < \frac{\pi}{3\sqrt{3}}$ show that for any $n \geq 3$ there exists such a polygon with $\frac{A}{B} = k$.