# PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompannied by your name, school and year or form. Solutions of these problems will appear in the next issue of Parabola; your solution(s) may be used if they are received in time.

**Q.782** Let $S_n = \frac{1}{1^2-\frac{1}{4}} + \frac{1}{2^2-\frac{1}{4}} + \frac{1}{3^2-\frac{1}{4}} + \cdots + \frac{1}{n^2-\frac{1}{4}}$

Simplify this expression, and show that when $n$ is large $S_n$ is approximately equal to 2.

**Q.783** In the following equation:-

$$908 + 47 + 26 + 13 + 5 = 999$$

the left hand side contains every digit exactly once. Either find a similar expression (involving only + signs) whose sum is 1000, or prove that it is impossible to do so.

**Q.784** In the equation $a_0 + a_1x + a_2x^2 + \cdots + a_nx^n = 0$ all the coefficients $a_0, a_1, \cdots, a_n$ are odd integers. If the equation has a rational root (i.e. there are integers $p, q$ such that $x = \frac{p}{q}$ satisfies the equation) prove that $n$ also must be odd.

**Q.785** The circumcircle of $\triangle ABC$ has radius not greater than 1cm. Show that the sum of the areas of squares with sides $AB, AC$ and $BC$ does not exceed $9cm^2$.

**Q.786** $\{a_1, a_2, a_3, \cdots, a_n, \cdots\}$ is an increasing list of positive integers. A second list $\{b_1, b_2, \cdots b_n, \cdots\}$ is defined by the rule:- $b_n$ is the number of members of the first list which are less than $n$.

Show that any positive integer $N$ may be expressed either as $N = a_n + n$ for some $n$ or else as $N = b_n + n$

(For example if $\{a_n\} = \{2, 4, 6, 8, \cdots, 2n, \cdots\}$ then $\{b_n\} = \{0, 0, 1, 1, 2, 2, 3, 3, \cdots\}$.

If $N = 9$, then it is expressible as $a_3 + 3$ but not as $b_n + n$ for any $n$.

If $N = 10$ it is expressible as $b_7 + 7$, but not in the form $a_n + n$.)

**Q.787** Let $a_1, a_2, \cdots a_n$ be any positive numbers.

If $P = (i_1, i_2, \cdots i_n)$ is any arrangements of $\{1, 2, 3, \cdots n\}$ let

$$x_P = \frac{1}{a_{i_1}(a_{i_1} + a_{i_2})(a_{i_1} + a_{i_2} + a_{i_3}) \cdots (a_{i_1} + a_{i_2} + \cdots + a_{i_n})}$$

Show that the sum of all $n!$ of the numbers $x_P$ is equal to $\frac{1}{a_1 a_2 \cdots a_n}$

(e.g. if $n = 2 \frac{1}{a_1(a_1+a_2)} + \frac{1}{a_2(a_2+a_1)} = \frac{1}{a_1 a_2}$. This is easy to check. You are asked to prove the corresponding result for any positive integer $n$.

**Q.788** Six dollar coins are arranged in a triangle (see Fig.1). Show how they may be brought into the circular formation in Fig.2. An allowable move consists in sliding one coin, without disturbing the others, to a new position in which it touches two others.

Fig 1

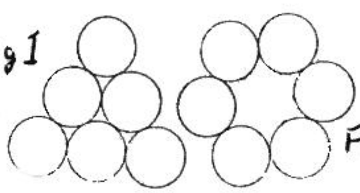

Fig 2

**Q.789** The sum of seven consecutive whole numbers is a perfect fifth power. If the smallest and the largest are deleted the sum of the remaining five is a perfect cube. Can you find such a set of integers?

**Q.790** Find the shortest distance from the point $A(0, a)$ to the arc of the parabola $y = x^2$ lying between $x = 0$ and $x = b$, for all numbers $a$ and $b$.

**Q.791** (i) Given any polygon, of area $A cm^2$, prove that it can be chopped up into a

(Continued on p.36)