

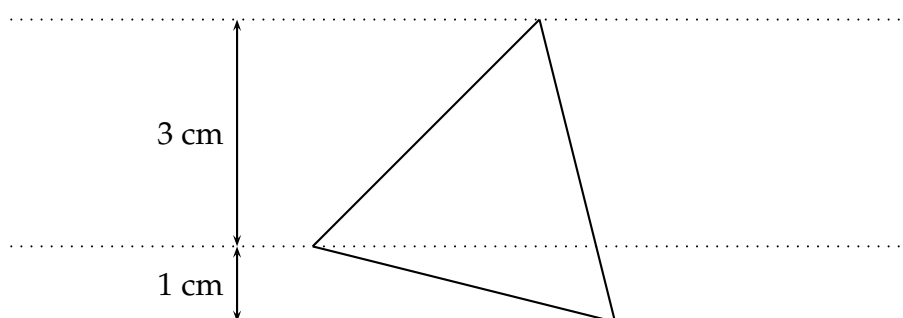
Problems 1321–1330

Q1321 Find the sum of the coefficients of those terms in the expansion of

$$(x^{31} + x^5 - 1)^{2011}$$

which have an odd exponent in x .

Q1322 The vertices of an equilateral triangle lie on three parallel lines which are 3 cm and 1 cm apart, as shown in the figure. Find the side length of the triangle.



Q1323 Let $f(x)$ be the sum of all terms in the expansion of $(x + a)^n$ which have an odd exponent in x , and let $g(x)$ be the sum of all terms which have an even exponent. Prove that

$$f^2(x) - g^2(x) = (x^2 - a^2)^n.$$

Q1324 Jack wrote down all integers from 1 up to 999, in exactly that order. After writing down 540 digits, he stopped for a break. Which digit was the last one written?

Q1325 An astronaut is at a point P in space. At this point, the earth and the moon appear to be equally large to him. Let E and M be the centres, and R and r be the radii of the earth and the moon, respectively. Find PE/PM in terms of R and r .

Q1326 Find the value of

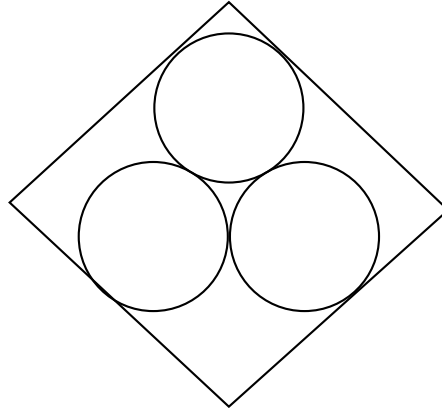
$$2010^2 - 2009^2 + 2008^2 - 2007^2 + \cdots + 4^2 - 3^2 + 2^2 - 1^2.$$

Q1327 A quadrilateral $ABCD$ contains an inscribed circle C . Assume that the ratio of the area of Q to that of S is $4/3$. Find the ratio of the perimeter of Q to that of C .

Q1328 Find the maximum and minimum values of

$$A = \sin^3 x + \cos^3 x.$$

Q1329 Three discs of radius 1 unit are contained in a square, as shown in the diagram. Find the length of the square.



Q1330 For any natural numbers m and n , let

$$S_m(n) = 1^m + 2^m + \cdots + n^m.$$

Prove that

$$\binom{m+1}{0} S_0(n) + \binom{m+1}{1} S_1(n) + \cdots + \binom{m+1}{m} S_m(n) = (n+1)^{m+1} - 1,$$

where, for $0 \leq k \leq m+1$,

$$\binom{m+1}{k} = \frac{(m+1)!}{k!(m+1-k)!}.$$