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Problems 1331–1340

Q1331 Given any positive integers m and n prove that every divisor of mn can be expressed as a product of a divisor of m and a divisor of n.

Q1332 Show that 7999999999999999 is not a prime number.

Q1333 Find the largest coefficient when $(x + 2y + 3z)^{99}$ is expanded and like terms collected.

Q1334 Suppose you have 100 lightbulbs numbered from 1 to 100, and that each lightbulb has a push button on/off switch. Initially, all lightbulbs are off. Now consider the following sequence of steps: In the first step, press the switches for those lightbulbs whose numbers are divisible by 1 (i.e. turn on all lightbulbs). In the second step, press the switches for those lightbulbs whose number is divisible by 2. Continue in this fashion and in the k-th step, press the switches for those lightbulbs whose number is divisible by k.

After 100 steps, which lightbulbs are switched on?

Q1335 What is the probability that two or more students in a class room of thirty share the same birthday?

Q1336 Two lighthouses stand a distance d km apart along the coast. A ship sails from one to the other, taking a course such that the angle between the lighthouses, as seen from the ship, is always α. What is the area of the largest triangle formed by the ship and the two lighthouses in the course of the voyage?

Q1337 On an $n \times n$ chessboard ($n \geq 2$) we wish to colour a 2×2 block of squares red and another 2×2 block blue. The red and blue blocks may not overlap. In how many ways can this be done?

Q1338 Given any two odd integers, a and b, prove that $a - b$ is divisible by 2^{2010} if $a^{2011} - b^{2011}$ is divisible by 2^{2010}.

Q1339 The cubic equation $x^3 - 2x^2 - 3x - 4$ has three solutions a, b, c, all different. Prove that

$$\frac{a^{2010} - b^{2010}}{a - b} + \frac{b^{2010} - c^{2010}}{b - c} + \frac{c^{2010} - a^{2010}}{c - a}$$

is an integer.

Q1340 ΔABC has side lengths $AB = 15$ cm, $AC = 14$ cm, $BC = 13$ cm. Its area is 84 cm^2. The point P lies closest to the side AB but outside the triangle. The distances from P to AB and AC are 3 cm and 8 cm respectively. Find the distance of P to BC.