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Problems 1341–1350

Q1341 A lazy weather forecaster predicts that future maximum temperatures will be the average of the preceding two days maximum temperatures. The forecaster starts his forecast by noting that yesterday's maximum temperature was 23^{o}C and the day before it was 29^{o}C. What is the weather forecaster's long term maximum temperature forecast?

Q1342 An employer is negotiating with a union on future staff salary increases. The union asks for an outcome with a flat salary increase of 18% at the end of 4.5 years. The employer offers two 2% pay rises each year with the first increase in January and the second in July. The Treasury estimates the CPI to be 3.5% per year for the next 4.5 years.

Does the employer's offer match the union request? Does the employer's offer match the CPI?

Q1343 Imagine that you stand at the origin of a number line and take a random walk along the line directed by the toss of a coin. If the coin shows heads you take a step to the integer on the left and if the coin shows tails you take a step to the integer on the right. After $2n$ tosses of the coin what is the probability that you will be at position n?

Q1344 A man is thirty years old. His sister is twice as old as he was when she was thirty years old. How old is the woman?

Q1345 Notice that the function

$$g(x) = \frac{1}{1-x}$$

has the properties

$$g(x) \neq x\,, \quad g(g(x)) \neq x\,, \quad g(g(g(x))) = x\,.$$

In a simpler notation we can write this as

$$g(x) \neq x\,, \quad g_2(x) \neq x\,, \quad g_3(x) = x\,.$$

Find real numbers p, q, r, s such that the function

$$h(x) = \frac{px+q}{rx+s}$$

has the properties

$$h(x) \neq x\,, \quad h_2(x) \neq x\,, \quad h_3(x) \neq x\,,$$
$$h_4(x) \neq x\,, \quad h_5(x) \neq x\,, \quad h_6(x) = x\,.$$

Q1346 (a) Find a function $f(x)$ defined for all real numbers $x \neq 0, 1$ such that the equation

$$f(x) + x^2 f\Big(\frac{1}{1-x}\Big) = x$$

is true for all x.

(b) Prove that the equation

$$f(x) + x^3 f\Big(\frac{1}{1-x}\Big) = x$$

has no solution.

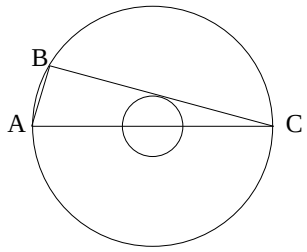
Q1347 On an 8×8 chessboard, how many ways are there to paint five of the squares red,

(a) if there must be a horizontal row of at least four consecutive red squares?

(b) if there must be a row (in any direction) of at least four consecutive red squares?

Q1348 Let p and q be consecutive primes other than 2. Show that $p + q$ is a product of at least three prime numbers (not necessarily all different).

Q1349 In the following figure the larger circle has a radius four times larger than the radius of the smaller circle. AC passes through the centre of the concentric circles. What is the length of the chord AB if the radius of the larger circle is 24?



Q1350 Alan, Betty, Chris and Debbie are of different ages. They all speak the truth whenever they talk to an older person, but they all lie when talking to a younger person.

> Alan says to Betty, "I am older than Chris". Debbie says to Chris, "You are the youngest of us all".

List the four people in order from youngest to oldest.