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Problems 1351–1360

Q1351 A city consists of a rectangular grid of roads, with m roads running east–west and n running north–south. Every east–west road intersects every north–south road. A construction vehicle travels around the city, visiting each intersection once (and only once) and finally returning to its starting point. As it travels it builds a fence down the middle of each road it uses: thus it constructs, in effect, a single long fence which eventually loops back on itself. How many city blocks are now inside the fence?

Q1352 Find infinitely many triangles with integer side lengths which contain an angle of $120°$.

Q1353

(a) On a $4 \times n$ chessboard we wish to place $2n$ knights in such a way that none attacks any other. Give three possible ways of doing this.

(b) Prove that there is no closed knight's tour on the $4 \times n$ board.

Q1354 A sequence $a_1, a_2, a_3, \ldots$ of positive integers has the properties

$$a_n^2 - a_{n-1}a_{n+1} = 1$$

for all $n \geq 2$, and $a_1 = 1$.

(a) Prove that a_2 cannot equal 1.

(b) Prove that if $a_2 = 2$ then $a_n = n$ for all n.

(c) Prove that if $a_2 \geq 3$ then $a_n > a_{n-1} + 1$ for all $n \geq 2$.

(d) Find all values of a_2 and all values of n such that $a_n = 2011$.

Q1355 Let n be a positive integer and suppose that 2^n and 5^n begin with the same digit. Prove that there is only one possibility for this digit.

Q1356 Let $N = 11!$ How many positive cubes divide N?

Q1357 Find the sum of the digits of the number

$$N = (2011^2)(625^{2011})(4^{4022}).$$

Q1358 Suppose that $\log(x^2y^{2n}) = 1$ and $\log(x^{2n}y^2) = 1$ where x and y are positive numbers and n is a positive integer. Show that

$$\log(x^ny^n) < 1.$$

Q1359 Simplify

$$\frac{1}{\sqrt{2}+\sqrt{1}}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{2011}+\sqrt{2010}}\,.$$

Q1360 Shown below is a map of walking paths around a garden. Paths meet at the points labelled A to M. Note that x is not a path intersection but a place where one path runs along a bridge over another. Bridget is standing on top of the bridge at x and wishes to take a walk around the garden, visiting every path intersection exactly once and returning to x. In how many ways can she do this?