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Problems 1361–1370

Q1361 Find a six digit number which can be split into three two digit squares and also into two three digit squares. (The first digit of a number cannot be zero.)

Q1362 Sandy leans a ladder against a wall in order to clean the gutter running along the top of the wall. Sandy is worried that the foot of the ladder is going to slip away from the wall and therefore ties a tightly stretched string between the middle of the ladder and a nail which is located directly below the top of the ladder, at the point where the floor meets the wall. Assuming that the floor is perfectly horizontal and the wall is perfectly vertical, how much is this going to help?

Q1363 Find the smallest possible value of $x^2 + y^2$, if x and y are real numbers for which $y \geq 2 + 3x$ and $y \leq 7\sqrt{x}$.

Q1364 Find the total surface area, and the volume, of a regular tetrahedron inscribed in a sphere of radius r.

Q1365 Show that $99999999999999999999999999999999991$ is not prime.

Q1366 Draw a rectangle $ABCD$ with side lengths $AB = CD = 4$ and $AD = BC = 5$. Let M be a point on BC with $BM = 1$, and N a point on CD with $CN = 1$. Use this diagram to prove that

$$\frac{\pi}{4} = \arctan\frac{1}{4} + \arctan\frac{3}{5} \ .$$

Q1367 If n is a positive integer and p is a prime, we write $\nu(p, n!)$ for the exact power of p which is a factor of $n!$: that is, p^ν is a factor of $n!$ but $p^{\nu+1}$ is not. For example, $\nu(3, 10!) = 4$ because 3^4 divides $10!$ but 3^5 does not. Prove that

$$\nu(p, n!) = \left\lfloor\frac{n}{p}\right\rfloor + \left\lfloor\frac{n}{p^2}\right\rfloor + \left\lfloor\frac{n}{p^3}\right\rfloor + \cdots \ .$$

Q1368 If $N = 2011!$, how many hundredth powers are factors of N?

Q1369 Prove that if

$$a + \sqrt{b} = c + \sqrt{d} \ ,$$

where a, b, c and d are rational numbers and $\sqrt{b}$ and $\sqrt{d}$ are irrational, then $a = c$ and $b = d$.

Q1370 Find the smallest positive integer a for which the surd

$$\sqrt{a + 20\sqrt{11}}$$

can be simplified as $\sqrt{x} + \sqrt{y}$, where x and y are positive integers.