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Problems 1371–1380

Q1371 Consider shuffles of a standard 52–card pack. (See the article in this issue for terminology and basic information.) Let rev be the shuffle which reverses the pack – that is, the first card is swapped with the last, the second with the second last, and so on. As in the article, out denotes the outshuffle.

(a) Write a formula for $rev(k)$ in terms of k; also, write rev as a product of cycles.

(b) Without any calculation, write down the cycle type of the composite shuffle $rev \circ out \circ rev$.

(c) Show that if we shuffle a pack of cards with both an outshuffle and a reverse shuffle, it makes no difference which one we do first.

Q1372 Show that a composition of $n-1$ cycles

$$(\,1\;2\,)\circ(\,1\;3\,)\circ(\,1\;4\,)\circ\cdots\circ(\,1\;n\,)$$

can be written as a single cycle. Is

$$(\,1\;n\,)\circ\cdots\circ(\,1\;4\,)\circ(\,1\;3\,)\circ(\,1\;2\,)$$

the same cycle? For any numbers $a_1, a_2, \ldots, a_m$, describe the shuffle

$$(\,a_1\;a_2\;\cdots\;a_m\,)\circ(a_m\;\cdots\;a_2\;a_1\,)\,.$$

Q1373 Find a natural cubic spline for the data points

$$(x,y) = (0,4)\,,\;(1,7)\,,\;(3,5)\,.$$

(See the article in this issue for what this means and how to do it.)

Q1374 Postman Pat has an infinite number of letters to deliver in Integer Avenue, which contains an infinite number of houses. Each letter is labelled with an integer (positive, negative or zero), and different letters have different labels. The letter with label n is to be delivered to house number $(4n+1)^2$. Prove that no house receives more than one letter.

Q1375 Find a triangle with integer side lengths, all different, such that the longest side is 2011 and one of the angles is $\cos^{-1}\frac{1102}{2011}$.

Q1376 Prove that any number of the form $n = 17272\ldots27271$ is a multiple of 19.

Q1377 Let a be a positive integer. Prove that if n is an integer and $n > 2a^2$, then $n! > a^n$.

Q1378 Suppose that x satisfies the equation

$$x^{10^{10^{10}}} = 10^{10^{10^{11}}} .$$

We would like to know how many decimal digits x has.

(a) What would you guess? Ten digits? A hundred? A million maybe?

(b) Calculate the actual answer.

Q1379 If

$$\begin{aligned} 123x + 456y + 789z &= a \\ 456x + 789y + 123z &= b \\ 789x + 123y + 456z &= c\,, \end{aligned}$$

find *with minimum possible hard work* x, y, z in terms of a, b, c.

Q1380 I form a list of numbers by sticking together two consecutive positive integers. Thus I start with $12, 23, 34$; a bit later I get $89, 910, 1011, 1112$; I keep on going until I reach $9798, 9899, 99100$. What is the sum of all these numbers?