**Parabola Volume 48, Issue 1 (2012)**

# Problems 1381–1390

**Q1381** It is commonly believed that the minute hand and the hour hand on a clock are in *exactly* symmetrical positions when the time is $10:08$ and $42$ seconds.

(a) Without detailed calculations, prove that this is wrong.

(b) When, at about this time, are the hands exactly symmetrical?

**Q1382** Which is bigger: the ratio of the volume of a sphere to that of its inscribed cube, or the ratio of the volume of a cube to that of its inscribed sphere?

**Q1383** Choose as many as possible of the integers $1, 2, 3, \ldots, 100$, subject to the following conditions:

- Every pair of your chosen numbers must have a common factor greater than $1$.
- There must be no number greater than $1$ which is a factor of *all* your chosen numbers.

**Q1384** For any real number $x$ we write $\lfloor x \rfloor$ for $x$ rounded to the nearest integer downwards, and $\lceil x \rceil$ for $x$ rounded to the nearest integer upwards. For example,

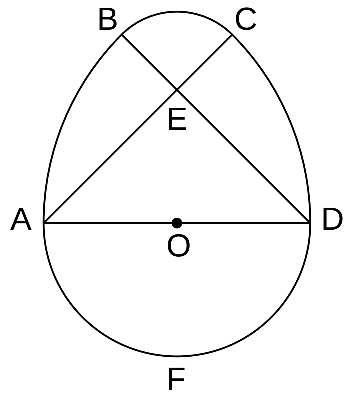
$$\lfloor \pi \rfloor = 3 \quad \text{and} \quad \lceil \pi \rceil = 4 \quad \text{and} \quad \lfloor 5 \rfloor = \lceil 5 \rceil = 5 \,.$$

Find all real numbers $x$ which satisfy the equation

$$\lfloor 6x \rfloor + 7x - \lceil 8x \rceil = 9 \,.$$

**Q1385** An $n$–digit integer is a string of $n$ decimal digits with no prohibition on leading zeros: for example, $005105$ is a valid six–digit integer. How many triples $(x, y, z)$ of $10$–digit integers are there which satisfy the conditions that $x, y$ and $z$ contain the digits $0, 1$ and $5$ only, and that $x + y = z$?

**Q1386** An egg shape is constructed as in the diagram. The line segment $AD$ has length $2$, and $O$ is its midpoint. The angles $\angle CAD$ and $\angle BDA$ are each $45°$. The curves $AB$, $BC$ and $CD$ are circular arcs with centres $D, E, A$ respectively, and $AFD$ is a semicircle with centre $O$. Find the area of the egg.




**Q1387** Find all solutions of the simultaneous equations in $2n$ variables.

$$\begin{aligned} x_1^2 + x_2 &= 1\,, & 2x_2 + x_3 &= 1 \\ x_3^2 + x_4 &= 1\,, & 2x_4 + x_5 &= 1 \\ &\vdots & &\vdots \\ x_{2n-1}^2 + x_{2n} &= 1\,, & 2x_{2n} + x_1 &= 1\,. \end{aligned}$$

**Q1388** Consider the sequence of numbers obtained by stringing together the digits of the positive integers, namely

$$1\,,\ 12\,,\ 123\,,\ 1234\,,\ 12345\,,\ 123456\,,\ldots$$
$$\ldots,\ 12345678910\,,\ 1234567891011\,,\ 123456789101112$$

and so on. Prove that three consecutive numbers in this sequence can never have any common factor (except for $1$).

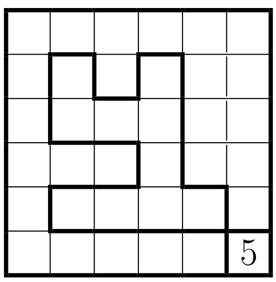
**Q1389** (inspired by the popular KenKen® puzzle (`www.kenken.com`)) Fill in one of the numbers $1, 2, 3, 4, 5, 6$ in each empty square of the diagram given below, in accordance with the following rules.

(a) Each horizontal row must contain the numbers $1$ to $6$, once each.

(b) Each vertical column must contain the numbers $1$ to $6$, once each.

(c) The numbers in the "inner" region of ten squares must have a sum of $26$.

(d) The numbers in the "outer" region of twenty–five squares must have a product of $24186470400000$.



**Q1390** I have a bag of counters labelled with the numbers $1$ to $2012$, once each. I am to choose counters from the bag, but I am not allowed to take any two counters whose numbers differ by $10$. What is the maximum number of counters I can take?