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Problems 1391–1400

Q1391 Jack looked at the clock next to his front door as he left home one afternoon to visit Jill and watch a TV programme. Arriving exactly as the programme started, he set out for home again when it finished one hour later. As he did so he looked at her clock and noticed that it showed the same time as his had done when he left home. Puzzling over how Jill's clock could be so wrong, Jack travelled home at half the speed of his earlier journey. When he arrived home he saw from his clock that the whole expedition had taken two hours and fifteen minutes. He still hadn't worked out about Jill's clock and so he called her up on the phone. Jill explained that her clock was actually correct (as was Jack's), but it was an "anticlockwise clock" on which the hands travel in the opposite direction from usual. Jack had been in such a hurry to leave that he hadn't noticed the numbers on the clock face going the "wrong" way around the dial. At what time did Jack leave home? (*Hint*: see the solution of problem 1381 in this issue.)

Q1392 Find all real numbers x which satisfy the equation

$$\lfloor x \rfloor - \{2x\} + \lceil 3x \rceil = 5\ .$$

As in problem 1384 we write $\lfloor x \rfloor$ for x rounded to the integer below, and $\lceil x \rceil$ for x rounded to the integer above; also, $\{x\}$ denotes rounding to the nearest integer, with halves rounding upwards. For example,

$$\{\pi\} = 3 \quad \text{and} \quad \{3\tfrac{1}{2}\} = 4\ .$$

Q1393 The sequence of numbers $a(1), a(2), \ldots$ in the solution to problem 1385 (later this issue) has the properties $a(1) = 5$, $a(2) = 26$ and

$$a(n) = 6a(n-1) - 4a(n-2) \quad \text{for} \quad n \geq 3\ .$$

Use the method of the article in this issue to find a formula for $a(n)$ directly in terms of n, and hence check the answer to problem 1385.

Q1394 Consider the problem of covering a hallway of width 3 metres with carpet tiles, if the tiles available are 2 metre by 2 metre squares and 2 metre by 1 metre rectangles. (See the article in this issue for a similar problem.)

(a) How many ways are there of doing this if the length (in metres) of the hallway is odd?

(b) Let $a(n)$ be the number of ways of carpeting a 3 metre by $2n$ metre hallway. Find a recurrence relation for $a(n)$ and solve it to obtain a direct formula for $a(n)$.

Q1395 Find all real values of a and b such that the parabolas

$$y = x^2 + ax + b \quad \text{and} \quad y = -x^2 + bx + a$$

do not intersect.

Q1396 Consider a triangle with sides of length a, b, c, and let l be the length of the line which bisects the angle opposite the side of length a. Prove that

$$l^2 = \frac{4bc}{(b+c)^2}\, s(s-a)\ ,$$

where s is the semi perimeter of the triangle. Deduce that the area of the triangle is at least

$$\frac{l_a l_b l_c}{8s}\ ,$$

where l_a, l_b, l_c are the lengths of the three angle–bisectors.

Q1397 Find a number consisting of four different digits in ascending order, such that if the digits are written in reverse to form another four-digit number, and if the two four–digit numbers are added, the result is 9218.

Q1398 Let p be a polynomial and suppose that $p(x)$ is a factor of $p(x^2)$. Prove that the equation $p(x) = 0$ has no real solutions except possibly 0, 1 or -1.

Q1399 There are n double seats in a railway carriage, all occupied. People get up and leave, one at a time, in random order. Find the probability that after the last pair is broken there remain exactly k people in the carriage.

Q1400 In the following diagram, in how many ways can one travel along the lines from point A to point B if one may only move upwards and to the right?