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History of Mathematics: The Foucault Pendulum

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The earth, as we all know, rotates about its axis once every twenty-four hours, giving us the cyclic alternation of night and day. An ingenious demonstration of this was designed in 1851 by the French scientist and mathematician Léon Foucault (1819-1868). The underlying idea is a very neat one, although its *realization* is actually quite problematic. It uses a pendulum. We are all familiar with the basic idea of a pendulum. One end of a string is attached to a fixed support (here referred to as the 'pivot'), the other to a weight (referred to as the 'bob'). The whole is suspended so that it hangs freely from the support. The string remains taut, but otherwise the bob is free to move. If it merely oscillates back and forth, its motion is confined to a plane and the pendulum is referred to as a 'simple pendulum'. Its behavior is analyzed in many school physics textbooks where the period of that oscillation is shown to be proportional to the square root of the length of the string.

As the simple pendulum moves in its plane, the bob is acted on by two forces: the tension in the supporting string and the pull of gravity. It is subject to Newton's laws of motion as it moves under the influence of these forces. Newton's laws apply in so-called 'inertial frames', or 'Newtonian frames'. Such frames are established in relation to the universe as a whole and are either at rest with respect to this or else moving in straightforward linear constant speed with respect to it.

If we look at the night sky, we see an unchanging pattern of stars. This pattern itself moves with the time of day and the seasons of the year, but in its internal composition it is unchanging. In fact, the pattern is said to be made up of the 'fixed stars'. This name is used to distinguish them from the planets (Greek: 'wanderers'), that move across the underlying pattern. Modern astronomy recognizes that the 'fixed stars' are not in fact fixed, but also realizes that their motions are almost imperceptible, because they are so very very far away. So it comes about that it is the remote reaches of our universe that are what defines the inertial frames.[2]

The plane of a simple pendulum is acted on by no 'sideways' force, and so preserves its orientation with respect to the 'fixed stars'. This is not however the same as saying that *we* would observe the pendulum as preserving its orientation. The most telling way to envisage what actually happens is to imagine a simple pendulum set up at the North Pole. An observer there would rotate with the earth, completing one circuit in

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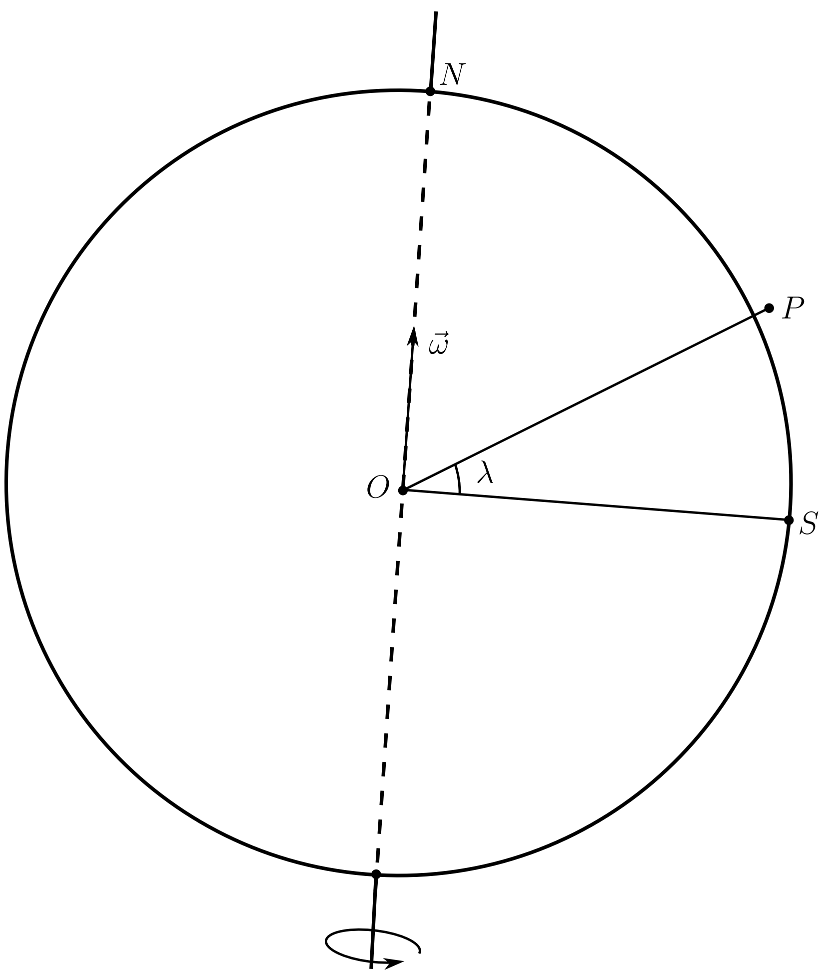
[2]It remains an ongoing puzzle why the laws of physics that we observe *locally* should be influenced by what goes on in the distant parts of the universe. Einstein referred to this connection as 'Mach's Principle' and attempted to incorporate it into his General Theory of Relativity. But it remains an enigma.

the course of a day, but from that observer's own point of view, it would seem as if the plane of the pendulum's swing was doing the rotating in the opposite direction and so it would appear to complete one clockwise circuit in the course of each 24 hours.[3]

The same phenomenon would be noticed at the South Pole, except that there the apparent motion would be anticlockwise. On the other hand, an observer at the Equator would see no effect whatsoever; this is because the pivot's own movement precisely matches that of the earth. Here I suppose this observer to be at Singapore, which is very nearly on the equator. At latitudes between these extremes (poles and equator), the effect is observed but is modified. To a very good approximation, the earth is a sphere. Refer to the diagram below, which shows a cross-section. O is the center of the earth, N is the North Pole, P the pivot (shown as somewhat above the surface of the earth), and S (for Singapore) a point on the Equator. The earth's angular velocity is a vector $\vec{\omega}$ with a magnitude usually denoted by ω and a direction along the earth's axis ON. The component of this experienced at P is $\omega \cos \angle NOP = \omega \sin \angle SOP = \omega \sin \lambda$, where λ is the latitude of P. Thus the magnitude of this component is $\omega \sin \lambda$.



[3] Actually this figure needs a slight adjustment. More exact is 23 hours, 56 minutes and 4.091 seconds. The difference is due to the earth's other motion: its revolution about the sun.

This same analysis also applies in the Southern Hemisphere, except that the apparent rotation will be reversed (anticlockwise instead of clockwise). So, for an observer at Grafton, whose latitude is 30° south, we must modify the period of the rotation. Because $\sin 30^\circ = 1/2$, the component of the earth's rotation is $\frac{\omega}{2}$ and in consequence, the pendulum would take two days to complete one circuit, and that circuit would be traversed in an anticlockwise direction.

[Another approach to the question is to use the 'terrestrial frame of reference' and to describe directly what an earthbound observer actually sees. Because such observers are located on the rotating earth, their frame of reference is not inertial and, because of that, Newton's laws of motion need modification. That modification requires the imposition of two further forces: the so-called 'centrifugal force' and the *Coriolis* force. The first of these is familiar to all of us. We experience it going around a corner in a car, for example. It is what makes a spin-dryer work etc. The Coriolis force is less familiar. It is named after Gustave Coriolis (1792-1843), a somewhat older contemporary and compatriot of Foucault. The force named in his honor deflects the path of an object moving in a rotating frame of reference. As a simple example, imagine a shell fired from a gun at the North Pole. Because the earth rotates beneath its trajectory, the point of impact will not lie exactly on the meridian along which it was fired, but a little to the west. It was only at the time of World War I that the effect became noticeable in gunnery practice, with the advent of more powerful and longer range ordnance. However, the concept of Coriolis force is widely employed in theoretical meteorology, explaining why the air circulates around those highs and lows you see on the weather map. If you Google **Foucault pendulum**, you will find that several websites explain the Foucault effect in terms of Coriolis force.]

Foucault first set up a pendulum demonstrating this effect in February 1851 in the Paris observatory, but shortly afterwards built another in Paris's *Panthéon* building. Since then many others have been built. For a list of some 300 or so (and even this is known to be incomplete), see the website
http://en.wikipedia.org/wiki/List_of_Foucault_pendulums

There are, however, considerable problems involved in obtaining reliable results from the motion of such pendulums. Many actual pendulums are in error by amounts that have been estimated as about 15%. Writing in February 1964, the *Scientific American* columnist Dr C L Stong had this to say: "None of the Foucault pendulums [I] have examined, including [and here he named two prominently displayed US examples] betters the 15% error". Since 1964, of course, there have been advances, and pendulums have become more accurate. In the rest of his column, Dr Stong went on to describe one that reduced the error to about 2%.

Indeed, even before that there were more accurate examples. Especially notable is one constructed and monitored by the Dutch physicist Heike Kamerlingh Onnes [4](1853-1926), who in 1879 wrote a doctoral dissertation incorporating his results. Using a short pendulum in a vacuum, he obtained results with only 1.5% error.[5]

[4]Kamerlingh Onnes was his surname; its two parts are sometimes hyphenated to emphasize this.

[5]Kamerlingh Onnes went on to a most distinguished career. In 1913, he was awarded the Nobel prize in Physics for his investigation of low temperature phenomena. He was the discoverer of superconduc-

In the years since Kamerlingh Onnes performed his experiments, there have been many Foucault pendulums constructed. The Wikipedia website given above gives details of many. It lists seven Australian examples, and I know of two that they don't mention; there may well be others. Those they do know about are one at UNSW, one in Canberra's Questacon Centre, one in Gingin WA, one at the University of Melbourne and another at Monash University (to be discussed below), one at the University of Adelaide and finally one at Brisbane's Museum Sciencentre. They miss a second example at Monash (in the Physical Sciences Building) and one at ABC Radio Sydney. This last was designed by the well-known popularizer of Science Karl Kruszelnicki. See
http://www.abc.net.au/science/kelvin/k2/pendulum/default.htm

A particularly interesting pendulum is one set up at the South Pole.

Foucault pendulums may be classified into two distinct types: the *free* and the *driven*. A free pendulum is simply that: a straightforward pendulum. Such a pendulum is well suited to experimental work such as that carried out by Kamerlingh Onnes, but is not always seen as well adapted to the demands of public display. The reason is that frictional effects will eventually bring the pendulum to rest. (This is called 'running down' .) Such a pendulum must be restarted at frequent intervals (and, as will appear below, this can be rather tricky). The driven pendulum has a supply of energy to keep it going, so that the need for constant restarting is eliminated. However, it is important to supply this energy in such a way that the Foucault effect is not thereby disturbed.

When a pendulum is set up for the purpose of experiment, then the question of running down is not especially important. However, if the principal purpose is display, then the preference is for a driven pendulum — although some displays use a free one and make a ceremony of starting it afresh each day!

Apart from the matter of running down, however, there are other problems confronting the would-be pendulum constructor. One is to design a pivot that does not itself affect the direction of swing. However the worst problem is that of 'ellipsing', to which I now turn.

If a pendulum is left to its own devices, it actually swings in a quite complicated manner. Its only constraint is the string that connects it to the pivot, and so it is only constrained to position its bob on the surface of a sphere of radius l, the length of the string. If we confine our attention to very small oscillations, then the path traced out by the bob is not necessarily straight (i.e. confined to one vertical plane), but actually elliptical (on the simplest of approximations). However this rough approximation misses some key factors. More detailed calculations show that the ellipse itself actually rotates, and the rate at which it does so is proportional to the area of the ellipse traced out by the bob. This rotation can be so pronounced that it completely masks or even cancels out the Foucault effect, so that the pendulum fails to behave as it is meant to.

The change in the orientation of the ellipse due to the Foucault effect is an angular

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tivity and was the first experimenter to liquefy helium.

displacement per swing of

$$\Delta\phi_F = 2\pi\omega\sin\lambda\sqrt{\frac{l}{g}}$$

(Here l is the length of the supporting string, g is the constant gravitational acceleration experienced at the earth's surface and the other symbols are either standard or previously introduced.)

The corresponding change brought about by the elliptical shape of the orbit is

$$\Delta\phi_E = \frac{3\pi ab}{4l^2}$$

(Here a is the semi-major axis of the ellipse, in other words half its 'length', and b is its semi-minor axis, in other words half its 'width'; the other symbols are as introduced before.)

It follows that, in order to demonstrate the Foucault effect, we need to eliminate the ellipsing, or at least greatly reduce it. For a start, it is necessary to release the pendulum from its starting position in such a way as to induce no sideways movement whatsoever and this is quite a difficult matter; the need to ensure that the bob swings precisely in one plane requires elaborate experimental procedures. But even this is not enough, because in a real-life situation there are always random disturbances that cause a sideways motion to be set up.

To show the effect of ellipsing on such a pendulum, take a hypothetical example. Consider a pendulum of length 10 metres set up at Grafton. Then $\sin\lambda = 0.5, g \approx 10$, $l = 10$, $2\pi\omega \approx 3.57 \times 10^{-4}$ (all in SI units) and so the Foucault is measured as $\Delta\phi_F \approx 2.28 \times 10^{-4}$. This is a very small number. In order to observe it accurately, we need to make $\Delta\phi_E$ even smaller (written $\Delta\phi_E \ll \Delta\phi_F$), and this is hard. For our hypothetical pendulum, we need $ab \ll 2.28 \times 10^{-4} \times 400/(3\pi) = 0.097$.

But in order to display the pendulum's motion, we can't reduce a too much. Suppose we allow the bob to swing through 80 cm, i.e. 40 cm either side of its lowest position, then we need $b \ll 0.097/0.4 \approx 0.24$. The sideways motion must be *much* less than 24 cm. So, even a one centimeter deviation either side ($b = 0.01$) will result in a percentage error of about 4%.

Thus, there have been various devices proposed in order to reduce the problem of ellipsing. The most common is the *Charron ring*, a narrow ring placed just below the pivot. It is widely believed that this minimizes, or even eliminates, the ellipsing problem. However, this belief is not universal and some authors still question it. All the same, this is the most commonly used device to address the problem. In many instances, the Charron ring is also used to deliver the power that must be supplied to a driven pendulum.

My attention was first drawn to the entire question of the driven Foucault pendulum by one of my own teachers, later a colleague at Monash University. During the mid-1970s, this man (Carl Moppert) and an engineering collaborator (Bill Bonwick) designed and built a Foucault pendulum in one of Monash's engineering workshops. Moppert designed the workings of the pivot and Bonwick a novel electromagnetic drive that supplied a force to maintain the pendulum's motion. It was so arranged that there was no sideways impetus on the bob; it was only nudged in the direction of its actual motion.

In early 1978, the prototype in the engineering workshop was replaced by a more ambitious structure installed in an otherwise unused liftwell in the Mathematics building. This was formally 'opened' by the Chancellor of the day (Sir Richard Eggleston) on June 16 of that year. In those early days of the Monash pendulum, an electronic record was kept of its performance. As efforts continued to improve its accuracy, various (at least three) different brakes were fitted in order to eliminate as far as possible any ellipsing. One 2-month run produced results so good that the pair claimed that this pendulum was the most accurate in the world. In the wake of this success, they installed another pendulum of the same design in the McCoy (Earth Sciences) building at the University of Melbourne, where it still occupies the main stairwell.

The first brake took the form of a sponge rubber ring attached to the circular frame that defined the extent of the motion. With this in place, the pendulum advanced $8.3°$ per hour, when it should have advanced $9.2°$. This was an error of almost 10% and indicated that the pendulum was retarded by the sponge rubber. The next attempt to prevent ellipsing involved an electromagnetic brake, but this was abandoned in favor of having the bob just touch the circular frame (from which the sponge rubber had been removed). It was this third configuration that claimed the world record for accuracy. In another experiment the pendulum was run without any anti-ellipsing device and the extent of the ellipsing observed. Bonwick claimed that if this effect was allowed for then the Foucault precession was reproduced to within an accuracy of 0.01%. But this was seen as 'cheating'!

Moppert died in 1984, and with his passing, the experimental phase of this pendulum's life ended, and the focus moved more towards display. Bonwick and I produced an account of the pendulum, its history and mechanism, and this was made available to those interested in learning more.[6]

In the intervening years, the appearance of further pendulums has probably made for accuracy greater than that achieved at Monash. However, the case is not entirely clear. Published records of the Monash pendulum's performance all predate the best run and the records of this one seem now to be lost. Nor are the performances of other pendulums all that easy to come by. I have heard it said that the Paris pendulum is the most accurate, but its *actual performance* does not appear on any of the websites I have been able to access. Certainly it has all the makings of an accurate pendulum. From the formulae for $\Delta\phi_F$ and $\Delta\phi_E$ given above, it follows that the longer the string, the more accurate the pendulum. The value of l for the Paris example is 67 m, a formidable figure.

Another possible claimant is the pendulum at the South Pole. See

[6]This account is now very hard to come by. Bonwick died in 2011. My copy seems to have been lost in one of the many moves I have had to make in the last ten years, and Monash's library (surprisingly) has none. I was able to consult one in the State Library of Victoria, which may well now hold the only copy in existence. In large measure, its text reproduces an article by Moppert and myself in *Function* (April 1982), but also includes technical detail supplied by Bonwick.

http://www.southpolestation.com/trivia/00s/southpolefoucault.html

In this case, $l = 33$ m, and the period is 24 hours and 50 minutes. The error is a little under 4%.

Sadly, our Monash pendulum has fallen on evil days. The display was modified in the first half of 2012, and in the process the anti-ellipsing mechanism was entirely removed. In the supposed interests of enhanced presentation, the pendulum itself was rendered inaccurate! [Regrettably this preference for style over substance, for 'spin' over information, goes well beyond today's Monash; it is widespread in many aspects of Australian public life.]

However, all is not lost. The Moppert-Bonwick pendulum at the University of Melbourne remains intact and incorporates the third, the most successful, of their anti-ellipsing schemes. The Earth Sciences department in whose (McCoy) building it is housed does not monitor its performance, but I checked it myself over one period of about 8 days. As best I could judge by eye, I estimated its advance at $9.25°$ per hour, for an error of about 2.6%, which seems pretty good, all things considered!