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Problems 1421–1430

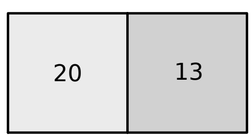
Parabola would like to thank Sin Keong Tong for contributing problem 1423.

Q1421 Find all integer solutions of the equation $2013x + 49y = 2$.

Q1422 Seven pirates are marooned on a deserted island. Before going to sleep they collect a pile of coconuts for food. During the night one of the pirates wakes up and decides to take his share. He divides the pile into seven equal parts and finds that there is one coconut left over, which he eats. He then puts six of the seven parts back into one pile and runs away with his share. Later two more pirates wake up. Not realising that one pirate has left, they also divide the pile into seven equal parts. There are two coconuts over; they eat these, put five shares back into one pile and run away with their own two shares. Later again three pirates wake up, divide the pile into seven, eat three leftover coconuts, take three of the seven shares and leave. The last pirate gets up in the morning. He sees that the other pirates have all gone, but he assumes they are out looking for more coconuts so he decides to share out those at the campsite. He finds that they divide into seven equal piles with none left over.

What is the smallest possible number of coconuts in the original pile?

Q1423 A rectangular dartboard has just two sections, which score 20 and 13 points respectively. With an unlimited number of darts available, is there a highest total not achievable? For instance, you can score $13, 20, 26, 33, 39, 40, 46$, but not any other total under 50.



Q1424 In problem 1413 (see the solution later this issue), suppose that we do not use all twenty–four numbers but remove those which are less than 1230. What is now the smallest possible value of

$$|x - 1230| + \cdots + |x - 2013| + \cdots + |x - 3210|\ ?$$

Q1425 As in problem 1414, an (a, b)–*superknight* is a knight–like chess piece which moves a squares up, down, left or right and then b squares in a perpendicular direction. In problem 1408 we saw that a $(3, 8)$–superknight can move from a square to the square immediately to its right in thirteen moves: we have

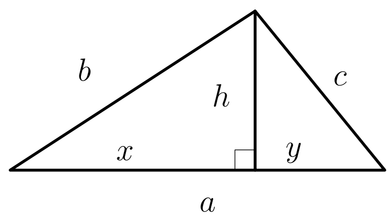
$$5(8, -3) + 3(-8, -3) + (-3, -8) + 4(-3, 8) = (1, 0)\ ,$$

and the number of moves on the left-hand side is $5+3+1+4=13$. Next question: is it possible to do this in fewer than 13 moves?

Q1426 The number 132 is a multiple of 1, and of 3, and of 2. What is the largest number you can find which has all its digits different, does not contain zero, and is a multiple of each of its digits?

Q1427 As in problem 1419, we wish to write a given number n as a sum of positive integers in such a way that the product of the summands is as large as possible. For this question, however, the summands must be 4 or more. For example, $14=4+5+5$ is allowed but $14=2+3+4+5$ is not. If $n=2013$, what is the maximum product we can obtain in this way?

Q1428 Use the following diagram to find the area of a triangle in terms of its side lengths a, b and c. (The answer is a formula which you may know already.)



Q1429 For what values of the coefficients a, b, c, d can the quartic equation

$$x^4+ax^3+bx^2+cx+d=0$$

be solved by using the method of problem 1412 (solution later this issue)?

Q1430 Find all solutions of the equation

$$x^2-12\lfloor x\rfloor+23=0\ ,$$

where $\lfloor x\rfloor$ denotes the integer part of x, that is, x rounded *down* to the nearest integer.