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Problems 1431–1440

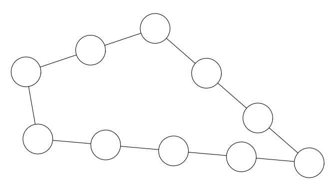
Q1431 Find a four–digit number with the following property: if the last digit of the number is moved to the front and 7 is added to the result, the answer is exactly twice the original number. Is there more than one such number?

Q1432 Use the method of mathematical induction (if you have not yet learned it at school, it is explained in Michael Deakin's article in this issue) to prove that every one of the numbers

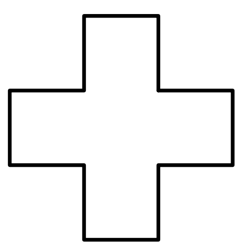
$$171\,,\ 17271\,,\ 1727271\,,\ 172727271\,,\ 17272727271\,,\ldots$$

is a multiple of 19.

Q1433 Fill in the circles in the diagram with the digits 0 to 9, in such a way that no two consecutive digits occur in adjacent circles, and the sum of the numbers on each side of the quadrilateral is the same.



Q1434 In the figure shown below all sides have equal length and each angle is a right angle. Show how to cut the figure into three pieces which can be rearranged to form a rectangle with sides in the ratio $2 : 1$.



Q1435 Adam, Betty, Cathy, Doug and Ellen have 10 coins, all of equal value, and are going to share them by the following procedure.

- Ellen will propose a distribution of the coins (so many to herself, so many to Doug and so on). The other four will vote on it. If 50% or more votes are in favour of Ellen's proposal it is accepted and the coins are distributed accordingly.
- If Ellen's proposal is rejected then she receives no coins and goes home. In this case Doug makes a proposal which is voted upon by the other three. Once again, if 50% or more are in favour then Doug's proposal is accepted; if not, Doug goes home with nothing.
- As long as proposals are rejected, the procedure continues, with one person fewer each time. If, eventually, Betty has a proposal rejected, then Adam gets all the coins.

How many coins will each person receive? We assume that all the participants are rational, and that they know the others are rational too. "Rational" means that they are able to work out the precise consequences of their actions; and that a person proposing a distribution of the coins will make the proposal which will bring him/her the maximum possible number of coins; and that each person will vote against a proposal if doing so *cannot* give him a worse outcome than voting for it, but will vote for it otherwise.

Q1436 We saw in problem 1430 (solution later this issue) that the equation $x^2-12\lfloor x\rfloor+23=0$ has four solutions, where the notation $\lfloor x\rfloor$ denotes x rounded to the nearest integer downwards. Show how to find positive integers a and b for which the equation

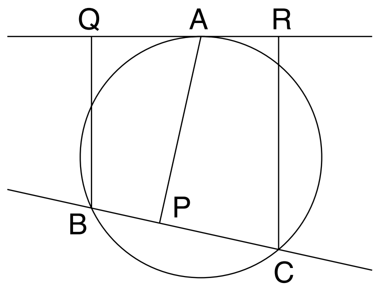
$$x^2-2a\lfloor x\rfloor+b=0$$

has as many solutions as desired. In particular, find an equation of this type which has more than 2013 solutions.

Q1437 As in previous questions, $\lfloor x\rfloor$ denotes x rounded to the nearest integer downwards. Prove that if we calculate the expressions n^2+n-1 and $n+\lfloor\sqrt{n}\rfloor$ for $n=1,2,3,\ldots$, we obtain all the positive integers once each.

Q1438 A line is tangent to a circle at the point A; another line cuts the circle at two points B and C; these two lines meet at M. Prove that $(MB)(MC)=(MA)^2$.

Q1439 In the diagram, the line QR is tangent to the circle at A; the angles at P,Q and R are right angles. Prove that $(BQ)(CR)=(AP)^2$.



Q1440 Let $f(x)$ be a polynomial with degree 2012, such that

$$f(1) = 1\ , \quad f(2) = \frac{1}{2}\ , \quad f(3) = \frac{1}{3}\ , \ldots , \quad f(2013) = \frac{1}{2013}\ .$$

Find the value of $f(2014)$.