Parabola Volume 60, Issue 2 (2024)

Opinion: Benefits of using base 8 over base 10 in society

Brian K. White [1] and **Edward T. Bednarz III**[2]

1 Introduction

The number system used by society is a very important decision that is rarely investigated. Society uses decimal (base 10) as the standard number system but here it is shown that octal (base 8) is a preferable number system. Other number systems are analysed and evaluated to show society's tendencies to categorise and organise into bases of the type 2^n; namely 2, 4, 8, and 16. This type of base is proposed to be called a *natural* base. While it is not anticipated that switching to octal is forthcoming in the near future, over the years, society has shown that investigating new ideas, even if not practical, can have long-term advantages that are not always predictable.

There are many different number bases that can be used by society. Base 2 is called binary, base 4 is quaternary, base 8 is octal, base 10 is decimal and base 16 is hexadecimal [1]. It is proposed to call any base number that is 2^n a natural base number, and the corresponding number system shall be called a natural base number system. Natural base number systems will be shown to be more in harmony with logic, the laws of mathematics, and natural human tendencies to categorise.

In this article, let the base of a number be represented by a subscript. For example, octal 12 is written as 12_8 [2]. Decimal numbers shall remain numbers with no subscript, except in instances where clarification is necessary.

Humans, nature and the basic laws which govern life on earth have a natural tendency to divide entities into natural-base numbered categories. This will be a pattern shown many times throughout this article. Humans do this at an unconscious level. Computers use binary for all calculations and operations. At higher levels, computer programmers use octal and hexadecimal. Computer and electrical engineers do this as a conscious choice; they know and recognise computer calculations are more logical when using natural base number systems.

[1] Brian White is a Mechanical Engineer with 30 years of experience. He has a passion for mathematics and philosophy.

[2] Edward Bednarz is an Associate Professor of Mechanical Engineering for Wilkes University in Wilkes-Barre, PA, USA.

2 Categorization by natural bases

An investigation of the widespread use of binary to categorize will yield astounding results. Consider all of the following examples where binary is utilised to categorise opposites: male and female, day and night, happy and sad, love and hate, and the flip of a coin - heads or tails. Consider mathematics: addition and subtraction, multiplication and division, or derivative and integral. Many more examples are readily found in mathematics and in all areas of life.

Sporting events and various games are another example of how humans divide and categorise according to natural base numbers. American football, for example, has 4 quarters (quaternary) and 2 halves (binary) in a game. A soccer contest consists of 2 halves. Competition in general is binary: team A versus team B. Most competitive sports keep records only of wins and losses.

There are many other examples of how humans have an unconscious tendency to divide entities into natural-base numbered categories. For directions, we use quaternary: North, South, East, West. Or for finer subdivisions for directions we use octal: North, Northeast, East, Southeast, South, Southwest, West, Northwest.

Carpenters use 1/2's, 1/4's, 1/8's, 1/16's, etc. as fractions. The same situation applies with machinists, mechanical engineers, architects, and civil engineers, at least in the US. Look in any US American machinist's handbook or engineering reference manual, and you will see the same; most sizes and dimensions use natural base fractions for sizes of objects. Piping, metal thicknesses, drilling and tapping sizes, dimensions with fractions, mill sizes, and standard hole sizes all are listed as 1/2's, 1/4's, 1/8's, 1/16's, etc.

Cooking in English-speaking countries usually consists of dividing ingredients into natural base fractions, with some examples being: 1/8 teaspoon vanilla, 1/4 cup flour, 1/2 cup sugar, and so on. A gauge on an American car reporting the amount of gasoline left in a tank is divided into eighths. In an US American deli, it is commonly requested to slice meats and cheeses in natural base fractions, with some examples being: 3/4 pound cheese, 1/2 pound turkey, etc.

As shown, categorizing using natural base numbers shows a tidiness and congruency that is not found in the decimal number system. Humans tend to follow this inclination by unconsciously dividing and categorizing into natural base numbers. Of natural base number systems, we argue that octal is the most convenient for society to use as a number system.

3 Natural base number systems

An example of a number in different bases is shown here. The number to the right represents that digit's value. The number to the left represents the base times that number. The next number to the left represents the base squared times that number and so forth. For example, 20_{10} is represented as 24_8 in octal; in quaternary, it is 110_4; in binary, it is 10100_2; and, in hexadecimal, it is 14_{16}.

While hexadecimal requires fewer characters, it requires new symbols to represent the numbers. In hexadecimal, the numbers are notated as follows:

$$1_{16},\ 2_{16},\ 3_{16},\ 4_{16},\ 5_{16},\ 6_{16},\ 7_{16},\ 8_{16},\ 9_{16}, A_{16},\ B_{16},\ C_{16},\ D_{16},\ E_{16},\ F_{16},\ 10_{16},\ \ldots .$$

In hexadecimal, the number 1000_{10} is represented as $3E8_{16}$. Hexadecimal would also require more numbers for humans to remember than is typically convenient. It has been proposed that the typical number of units that a person can distinguish at a time is 6–8; see [2, 3].

The octave in music is a very interesting and applicable study in natural base usage. The frequency of vibration specifies a note in music. An octave uses a ratio of 2:1 in frequencies. This occurs when going from one note (for example middle C) to the same note one octave up. C one octave up is two times the frequency of middle C. The next note C down from middle C would be 1/2 the frequency of middle C. For any given starting point (not just for note C, but for any note) the ratios are as follows:

$$1 : 2\ :\ 4\ :\ 8\ :\ 16\ :\ 32\ :\ 64\ :\ 128\ :\ 256\ :\ 512\,.$$

Here, we see the use of natural base numbers to lay out the notes in a musical instrument (in this example, the piano) which we shall see repeatedly. If the octal number system were used for laying out these notes, then the frequency ratios would be as follows:

$$1_8\ :\ 2_8\ :\ 4_8\ :\ 10_8\ :\ 20_8\ :\ 40_8\ :\ 100_8\ :\ 200_8\ :\ 400_8\ :\ 1000_8\,,$$

and so on. Each successive note's frequency ratio is a number of the quantity 2^n.

Sporting tournaments with single elimination are broken down so that each winning team plays the same number of games, while other teams are eliminated. In tennis, grand slam tournaments begin with 128 players. After each subsequent round, half are eliminated. In the decimal system, the number of players per round are as follows:

$$128 : 64\ :\ 32\ :\ 16\ :\ 8\ :\ 4\ :\ 2\ :\ 1\,.$$

With the octal number system it would be broken down as follows:

$$200_8\ :\ 100_8\ :\ 40_8\ :\ 20_8\ :\ 10_8\ :\ 4_8\ :\ 2_8\ :\ 1_8\,.$$

The pattern is much easier to recognise for octal and can easily be continued much higher without a calculator to aid someone.

Binary and quaternary number systems are not practical, as they require a large number of characters to represent a given number [2]. In binary, the number 1000_{10} is represented as 1111101000_2. In quaternary, 1000_{10} is represented as 33220_4. In Octal, 1000_{10} is represented as 1750_8. Octal requires only about 10% more characters than decimal on average.

4 Data storage

As it has been shown, raising the number 2 to a power is very common throughout mathematics and day-to-day life. One obvious drawback to doing so in base 10 is how hard it gets to perform this calculation because the numbers never repeat in a pattern. In base 8, they always cycle between 1, 2 and 4 followed by zeroes. Another way to represent this is to look at how much information is stored to completely capture the number. A comparison between base 10 and base 8 is shown in Table 1. In base 10, the number of significant figures is constantly increasing. A huge benefit to base 8 is that the number of significant figures is always one. This would cut down on the space required for data storage.

Binary to decimal conversion is done in many areas of working with computers. Computers use binary and humans use decimal. Computers must convert between the two bases to operate, requiring complex calculations. Binary to octal conversion would be much less complex.

Table 1: Significant figures for 2^n

$\mathbf{n_{10}}$	$\mathbf{2^{n_{10}}}$	Significant Figures [base 10]	$\mathbf{n_8}$	$\mathbf{2^{n_8}}$	$\mathbf{2^{n_8}}$	Significant Figures [base 8]
0	1	1	0	1	1 [E8] + 0	1
1	2	1	1	2	2 [E8] + 0	1
2	4	1	2	4	4 [E8] + 0	1
3	8	1	3	10	1 [E8] + 1	1
4	16	2	4	20	2 [E8] + 1	1
5	32	2	5	40	4 [E8] + 1	1
6	64	2	6	100	1 [E8] + 2	1
7	128	3	7	200	2 [E8] + 2	1
8	256	3	10	400	4 [E8] + 2	1
9	512	3	11	1000	1 [E8] + 3	1
10	1024	4	12	2000	2 [E8] + 3	1
11	2048	4	13	4000	4 [E8] + 3	1
12	4096	4	14	10000	1 [E8] + 4	1
13	8192	4	15	20000	2 [E8] + 4	1
14	16384	5	16	40000	4 [E8] + 4	1

5 Natural base fractions

One of the greatest advantages of the octal numbering system occurs when considering fractions. As mentioned previously, engineers in the US typically use natural base division when creating drawings. However, machinists in the US typically only require 3 decimal places. Therefore, when considering a fraction such as 11/64, which has a decimal equivalent of 0.171875, this would be rounded to a value of 0.172. Information is lost in the conversion. With the proposed "octimal" system for octal fractions as shown in Table 2, all information is retained with only 2 octimal points. The value of 11/64 in base 10 would be 13/100 in base 8, or an octimal equivalent of 0.13.

Table 2: Decimal and Octal Equivalents

Decimal Fraction	Decimal Equivalent	Octal Fraction	Octal Equivalent
1/64	0.015625	1/100	0.01
1/32	0.03125	1/40 (2/100)	0.02
3/64	0.046875	3/100	0.03
1/16	0.0625	1/20 (4/100)	0.04
5/64	0.078125	5/100	0.05
3/32	0.09375	3/40 (6/100)	0.06
7/64	0.109375	7/100	0.07
1/8	0.125	1/10 (10/100)	0.10
9/64	0.140625	11/100	0.11
5/32	0.15625	5/40 (12/100)	0.12
11/64	0.171875	13/100	0.13
3/16	0.1875	3/20 (14/100)	0.14
13/64	0.203125	15/100	0.15
7/32	0.21875	7/40 (16/100)	0.16
15/64	0.234375	17/100	0.17
1/4	0.25	2/10 (20/100)	0.20

6 Conclusions

Using an octal numbering system would lead to many advantages for machinists, engineers, carpenters and others working frequently with fractions or multiples of two. Educational mathematics could provide an easier level of understanding for students. It would also be beneficial in everyday life ranging from cooking to sports.

Society may not switch over to the octal number system anytime soon. However, there are advantages to understanding the benefits of octal and the inherent harmony of natural base number categorisation. It would be possible in school curricula to teach different number bases to make future generations aware of these benefits. Greater knowledge of these advantages may lead to other discoveries and applications not covered in the scope of this article. This is quite possible without society making a switch to octal.

It is one of the primary intents of this article to make the reader more aware of the many examples of how humans unconsciously divide and categorise according to natural bases. Indeed, with a heightened awareness of this unconscious tendency, we will be more aware of our choices when we have the opportunity to choose how to divide and categorise new areas. With this awareness, we will be better prepared for new inventions and areas of discovery in our lives.

Acknowledgements

Thank you for assistance to M.K. Saunders, K. White, F. Capra, S. Warres, R. Nathan, R. Decowsky, A. Kapolka and J. Patel.

References

[1] E.W. Weisstein, *Base*, MathWorld, last accessed 2024-08-12.

[2] A.R. Planz, Adoption of the octal number system, *IEEE Transactions on Computers* **20** (1971), 593–597.

[3] C.T. Morgan, *Introduction to Psychology*, 2nd ed., McGraw-Hill, New York, 1961.