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Book review of *Vector: A Surprising Story of Space, Time, and Mathematical Transformation*

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Before reading the book under review — *Vector: A Surprising Story of Space, Time, and Mathematical Transformation*[2] by Robyn Arianrhod (UNSW Press, July 2024) — its title brought to mind my favourite foreword[3]:

> "[...] *is not a subject about which the average man would wax enthusiastic, nor is it one about which he knows much about* [...] *this humdrum, ordinary, but very useful subject* [...]"

I like linear algebra and have loved teaching it to thousands of students, and yet I could not imagine anyone *waxing enthusiastic* about simple vectors.

This book expanded my imagination by showing how even the simplest of concepts can have a fascinating and far from simple history. In particular, this book provides a historic account of how vectors, and later tensors, arose from a tangled and surprisingly interesting history of developing concepts that began thousands of years ago. What might seem simple and obvious to us now was not always so. In fact, it took many conceptual breakthroughs — and many academic feuds — before vectors could be conceived, understood and, finally, accepted.

From a different perspective, *Vector* can be seen as a history of the discovery of the perceived natural laws of physics during the past few thousands of years. The focus on vectors and tensors provides an interesting and well-chosen framework for this history, as does the focus on its pivotal figures. Their research, insights and opinions, and their interactions, are related in the book through excellent and well-written narratives that carry the reader smoothly and assuredly from chapter to chapter.

From the ancient Mesopotamians, Egyptians, Greeks, Chinese, Indians to the Arabian and European mathematicians, the Prologue and Chapter 1 sweep through a traditional telling of the present "modern" era of maths, with brief but nice nods to less-often mentioned cultures and to pre-modern maths such as that here in Australia. This sweep of history takes us to where the main story of the book begins, namely with 1619 and 1631 works by Thomas Harriot that included central notions of vectors and which, perhaps for the first time, fully embraced symbols as the language for maths.

This historical prelude serves partly to show how tables of data slowly evolved into mathematical objects in their own right. More importantly, it serves to illustrate what might be the main message of the book, namely that the use of symbols and other

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[2]An extract of this book can be found as another article in this issue of *Parabola*.

[3]From the book *Rapid Calculations* by A.H. Russell (6th ed., The Gregg Publishing Company, Ltd., London, 1946).

compact notation not only make maths easier to conduct; it allows deeper insight by its own appearance alone. This point is made clearly and convincingly throughout the book, with historical examples showing how researchers could identify patterns and ideas in mathematical expressions and equations, allowing them to discover deep insight, conceive new concepts and to discern behaviour relating seemingly unrelated objects.

Two striking examples are insights by two of the main figures in the book, James Clerk Maxwell and Albert Einstein. From the shape of his famous field equations, Maxwell saw that light propagated as an electromagnetic wave and that other types of such radiation might exist too; this was later confirmed by the discovery of radio waves. From his mathematical work on relativity theory, Einstein derived his famous equation $E = mc^2$ and a little later realised its profound implication that matter and energy are intrinsically related.

The cast of characters in the book's story is long and relatively diverse, and nicely includes many women who played prominent roles but who have often been ignored in previous tellings of the story. Several of these women, like Emmy Noether, were pioneers of research; others, such as Mary Somerville, were arguably just as influential through their public disseminations, summation and commentary on existing work.

Aside from the inclusion of women, *Vector* is a mostly traditional presentation of science history, carried by the tales of (Western) individuals and by the sometimes explicit promotion of scientific discovery. The ambient cultural forces and ideas are nearly invisible and do not feature among the story's cast. Nevertheless, *Vector* does a very good and helpful job of showing how researchers of maths find their insights by slow increments, working together with others and often needing to revise and improve their findings. Rome wasn't built in a day, let alone built by one person. It is also very good to read how many researchers' important insights are not accepted by research communities at the time of their conception; nor are the researchers necessarily given credit or respect.

These are very valuable points to illustrate, especially in the field of maths, dominated so often by the myth of the lone (white, male) genius who, in a flash of divine inspiration, shouts "Eureka!" and uncovers a new mystery of the universe. This heroic mythology is popular and can attract some to the pursuit of maths but it alienates far more people, and make many of those people feel stupid, unfit and like they don't belong among "real" mathematicians. As a lecturer, it breaks my heart to see so many students feel like this, and not few of my accomplished colleagues too, and I fight hard to make people see that they too can excel and even enjoy maths. I am very grateful to Robyn Arianrhod for helping to dispel this incorrect and harmful myth.

With respect to composition, Robyn Arianrhod writes very well, and admirably maintains a difficult balance between popular non-fiction and academic history of physics and mathematics. Robyn Arianrhod also maintains the equally difficult balance between writing a flowing story and providing important side-stories and explanations. These balances are facilitated through extensive endnotes, as well as an excellent and very helpful timeline of events. The endnotes provide a wealth of detail for readers in need of mathematical help in want of personal or historical trivia, as well as for experts interested in deeper mathematical and physical explanations.

It was very interesting to read about how vectors and vector spaces arose. However, I felt that Chapter 11 circled around tensors without arriving at an explicit and clear explanation of what they were. This was a little anti-climatic after the chapters building up to the arrival of tensors. Simple and explicit examples might better have helped readers' understanding. This is true in a few other parts of the book as well, and pictures could have been drawn more prettily.

These are however minor quibbles, and *Vision* is overall a beautiful book to read. It is very carefully researched and well written, and it is bound to make you think. Whether you are interested in popular history of science or would like to know more about the details of how vectors and the laws of physics developed together, I recommend this book.